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Influence of Adaptation Gain and Reference Model Parameters on System Performance for Model Reference Adaptive Control

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Abstract—This article presents a detailed analysis and comparative performance evaluation of model reference adaptive control systems. In contrast to classical control theory, adaptive control methods allow to deal with time-variant processes. Inspired by the works [1] and [2], two methods based on the *MIT rule* and *Lyapunov rule* are applied to a linear first order system. The system is simulated and it is investigated how changes to the adaptation gain affect the system performance. Furthermore, variations in the reference model parameters, that is changing the desired closed-loop behaviour are examinded.

Keywords—Adaptive control systems, Adaptation gain, MIT rule, Lyapunov rule, Model reference adaptive control.

I. Introduction

ADAPTIVE Control techniques allow to control systems where certain system parameters are not known or change over time, for example in aircrafts [3] or for control of digital servo motors [4]. This article discusses first order systems of the class

$$Y(s) = G(s)\,U(s) = \frac{b}{s+a}U(s)\ . \tag{1}$$

It is assumed that both the system gain b as well as the time constant a are unknown or time variant. Despite these limitations a controller is to be found which will achieve desired closed-loop dynamics.

One particular way of handling this problem is the technique of “Model Reference Adaptive Control” (MRAC). This involves the definition of a *reference process* model whose dynamics in response to a *reference input* should be followed by the *plant process*. For the plant process with unknown parameters, a specific *control law* alters the reference input signal in order for the plant's output signal to match the one of the reference model. This control law features time dependent controller parameters θ which reflect the algorithm's adaptation to the given plant system. The adaptation or learning component is incorporated by a time differential equation, the *update law*.

The reference model defines the desired system dynamics and

$$Y_m(s) = G_m(s)\,R(s) = \frac{b_m}{s+a_m}R(s) \tag{2}$$

produces a model output y_m. From the plant process

$$Y_p(s) = G_p(s)\,U(s) = \frac{b}{s+a}U(s) \tag{3}$$

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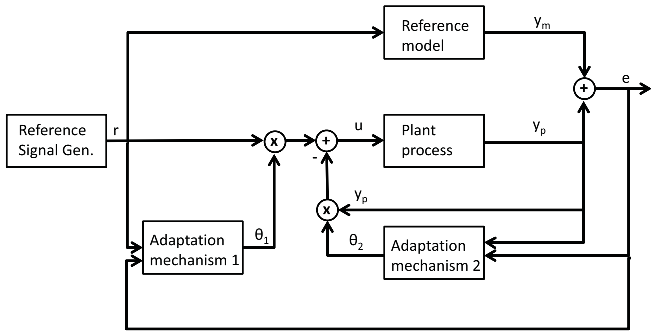

Fig. 1. Block diagram of a first order model reference adaptive control system.

y_p results. Taking the difference of these two signals yields the tracking error e.

The process dynamics

$$\dot{y}_p + a y_p = b u \tag{4}$$

should follow the reference dynamics

$$\dot{y}_m + a_m y_m = b_m r\ . \tag{5}$$

Chosing the *control law*

$$u = r\theta_1 - y_p\theta_2\ . \tag{6}$$

and inserting into eq. (4) yields

$$\dot{y}_p + (a + b\theta_2)\,y_p = b\theta_1 r\ . \tag{7}$$

The system structure using this control law is illustrated in the block diagram in fig. 1.

Indeed, if θ_1, θ_2 were to be found as

$$\theta_1 = \frac{b_m}{b} \tag{8}$$

and

$$\theta_2 = \frac{a_m - a}{b} \tag{9}$$

then eq. (7) would be identical to the reference dynamics in eq. (5).

However, as the true values a and b are unknown, adaptation mechanisms for θ_1, θ_2 which are solely based on measurable quantities have to be found. Two different *update laws* will be derived and experimentally evaluated in the following sections.

Similar work has been presented by the authors of [1] and [2]. This article enhances their approach with further experiments which analyse the effect of changes in the reference

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model parameters. These experiments are conducted using the MIT rule and Lyapunov rule and comparing the two methods.

The remainder of this article is organised as follows: First, the mathematical foundations of both update laws are outlined. Subsequently, experiments on the effect of adaptation gain are performed using the two algorithms. This includes analysis of the time response, output error and parameter estimation. Then, the reference model parameters a_m and b_m are varied and the resulting behaviour is analysed. Finally, a conclusion summarizes the results.

II. MIT RULE

Firstly, a cost function is defined:

$$J = \frac{1}{2} e^2. \tag{10}$$

The main idea is to change θ along the "steepest descent" of this cost function. Therefore, the time derivative is proportional to the negative gradient with an adaptation gain α:

$$\frac{d}{dt}\theta = -\alpha \frac{d}{d\theta} J \tag{11}$$

$$= -\alpha e \frac{d}{d\theta} e(\theta). \tag{12}$$

In a second step, the *sensitivity derivative* $\frac{d}{d\theta} e(\theta)$ is calculated. Inserting the control law defined in eq. (6) into the plant process dynamics and using $s(\cdot) := \frac{d}{dt}(\cdot)$ as differential operator yields:

$$y_p = \frac{b\theta_1}{s + a + b\theta_2} r. \tag{13}$$

It follows that the sensitivity derivatives $\frac{d}{d\theta} e(\theta)$ are given as:

$$\frac{d}{d\theta_1} e = \frac{d}{d\theta_1}(y_p - y_m) \tag{14}$$

$$= \frac{d}{d\theta_1}\left(\frac{b\theta_1}{s + a + b\theta_2} r - y_m\right) \tag{15}$$

$$= \frac{b}{s + a + b\theta_2} r. \tag{16}$$

And similarly:

$$\frac{d}{d\theta_2} e = \frac{d}{d\theta_2}(y_p - y_m) \tag{17}$$

$$= \frac{d}{d\theta_2}\left(\frac{b\theta_1}{s + a + b\theta_2} r - y_m\right) \tag{18}$$

$$= -\frac{b^2\theta_1}{(s + a + b\theta_2)^2} r \tag{19}$$

$$= -\frac{b}{s + a + b\theta_2} y_p. \tag{20}$$

However, eq. (16) and (20) still contain the unknown true parameters a and b. Therefore, an approximation has to be made for these terms. *Perfect model following* is achieved by chosing [5]

$$\theta_1 b = b_m \tag{21}$$

$$a + b\theta_2 = a_m \tag{22}$$

as it has already been indicated in the introduction.

Inserting these approximations into the sensitivity derivatives yields

$$\frac{d}{d\theta_1} e = \frac{b}{a_m} \frac{a_m}{s + a_m} r \tag{23}$$

and

$$\frac{d}{d\theta_2} e = -\frac{b}{a_m} \frac{a_m}{s + a_m} y_p. \tag{24}$$

Finally, inserting these expressions into the the approach (12) and merging factors into a single adaptation gain $\gamma = \alpha \frac{b}{a_m}$ as in [6] gives the update laws for θ_1 and θ_2:

$$\frac{d}{dt}\theta_1 = -\gamma e \frac{a_m}{s + a_m} r \tag{25}$$

$$\frac{d}{dt}\theta_2 = \gamma e \frac{a_m}{s + a_m} y_p. \tag{26}$$

III. LYAPUNOV RULE

In order to derive an update law using Lyapunov theory, the following Lyapunov function is defined [5], [6]:

$$V = \frac{1}{2}\gamma e^2 + \frac{1}{2b}(b\theta_1 - b_m)^2 + \frac{1}{2b}(b\theta_2 + a - a_m)^2. \tag{27}$$

The time derivative of V can be found as

$$\dot{V} = \gamma e \dot{e} + \dot{\theta}_1 (b\theta_1 - b_m) + \dot{\theta}_2 (b\theta_2 + a - a_m) \tag{28}$$

and its negative definiteness would guarantee that the tracking error converges to zero along the system's trajectories.

Inserting the dynamic equations of plant process (4) and reference model (5) yields

$$\begin{aligned}\dot{V} &= \gamma e(\dot{y}_p - \dot{y}_m) + \dot{\theta}_1 (b\theta_1 - b_m) + \dot{\theta}_2 (b\theta_2 + a - a_m) \\ &= -\gamma a_m e^2 + \left(\gamma e r + \dot{\theta}_1\right)(b\theta_1 - b_m) + \\ &\qquad \left(\dot{\theta}_2 - \gamma e y_p\right)(b\theta_2 + a - a_m). \end{aligned} \tag{29}$$

The second line gives the following condition for negative definiteness and thus the update laws:

$$\frac{d}{dt}\theta_1 = -\gamma e r. \tag{30}$$

$$\frac{d}{dt}\theta_2 = \gamma e y_p. \tag{31}$$

Note that this result is similar to the MIT rule in (25) and (26). The only difference is that the MIT rule comprises an additional filter operation with the reference model dynamics.

IV. EXPERIMENTAL PERFORMANCE EVALUATION

This section details an experimental performance evaluation of the adaptive controller. It is investigated how different adaptation gain values affect the system behaviour, both for the MIT rule and the Lyapunov adaptation strategy. Secondly, changes in the desired system response as defined by the reference model's parameters a_m, b_m are investigated.

Analysing the adaptive controllers is performed using a square wave signal with a time period of 20 s and unit amplitude. This can be interpreted as a repeated step response with each step lasting for a duration of 10 s. While it is common to use a single step function to examine ordinary

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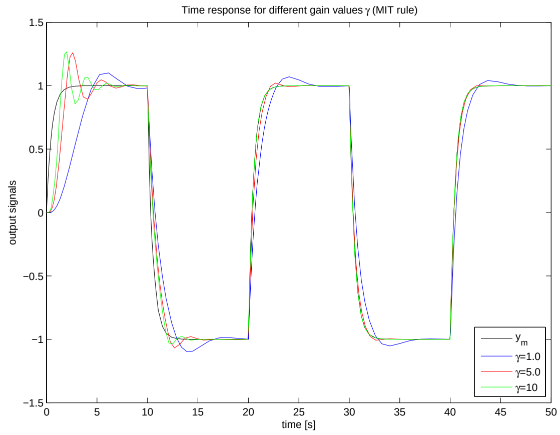

Fig. 2. MIT rule: Time response for varying adaptation gain values γ.

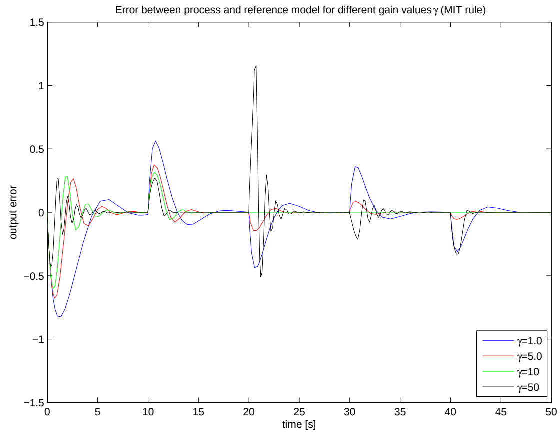

Fig. 3. MIT rule: Tracking error for varying adaptation gain values γ.

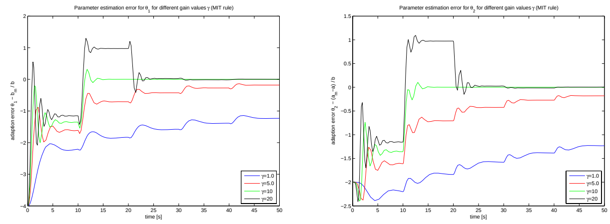

Fig. 4. MIT rule: Parameter estimation error for θ_1 (left) and θ_2 (right) for varying adaptation gain values γ.

TABLE I
MIT RULE: SETTLING TIME AND OVERSHOOT OF FIRST STEP INPUT FOR VARYING ADAPTATION GAINS (APPROXIMATIONS FROM FIG. 2).

	overshoot (1. step)	peak time (1. step)	5% settling time (1. step)
y_m (reference)	-	-	1.48 s
$\gamma = 1.0$	10.7%	5.79 s	7.04 s
$\gamma = 5.0$	26.2%	2.55 s	4.52 s
$\gamma = 10$	27.5%	1.89 s	4.22 s

closed-loop behaviour, the nature of *adaptive* systems makes it convenient to employ square wave signals and observe how the controller adapts over time.

In all cases, the true plant parameters were set to $a = 1.0$ and $b = 0.5$.

A. *Influence of adaptation gain using MIT rule*

At first, the adaptation gain γ is varied and the resulting influence on the system's time response is analysed. The results of an experiment with gain values 1.0, 5.0 and 10 are shown in fig. 2. Moreover, table I gives the approximate overshoot and 5% settling time as observed in these simulations.

The following observations can be made from these results:

- Increasing the gain value reduces the settling time. Thus it takes less time for the plant's output signal to approach the desired reference model's response.
- At the same time, oscillations and overshoot occur which are not present in the reference model's output (first order system). Increasing the gain value increases the frequency of these oscillations and also results in higher overshoot.

The tracking error between the reference system and the process over time is displayed in fig. 3. As the time responses already indicated, higher gain values result in shorter settling time for the error signal. This comes at the expense of higher overshoot and may cause undesired behaviour that is the adaptation may not converge. Analysing the gain margins for adaptive controllers has been subject of research, for example [7].

It is then investigated how the parameter estimates θ_1 and θ_2 relate to the ideal values $\theta_1 = \frac{b_m}{b}$ and $\theta_2 = \frac{a_m - a}{b}$. It can be seen from fig. 4 that the estimation error decreases over time while the controller "adapts" to the system. Although higher gain values may reduce the time needed for the parameter estimate to converge to the true value, this may also cause overshoot or even no convergence.[1]

In conclusion, it depends on the specific application's requirements whether a smaller overshoot or shorter settling times are favoured. In any case, the possibility of unwanted instability when increasing γ needs to be considered.

B. *Influence of adaptation gain using Lyapunov rule*

As in the previous section, adaptation gain γ is varied firstly and the resulting influence on the system's time response is examined. Conducting the same experiment as in previous section with gain values 1.0, 5.0 and 10 gives the results which are visualised in fig. 5. Furthermore, table II states the approximate overshoot and 5% settling time.

The tracking error between the reference system and the process over time is displayed in fig. 6. Similar to the the MIT rule results higher gain values result in shorter settling time for the error signal. But in contrast to fig. 3 no excessive overshoot can be observed when setting $\gamma = 50$.

Secondly, the parameter estimation error for θ_1 and θ_2 is recorded and shown in fig. 7. As with the results obtained

[1]Setting $\gamma = 50$ results in no convergence at all, in order to ensure readability of the plots, these graphs are is not depicted here.

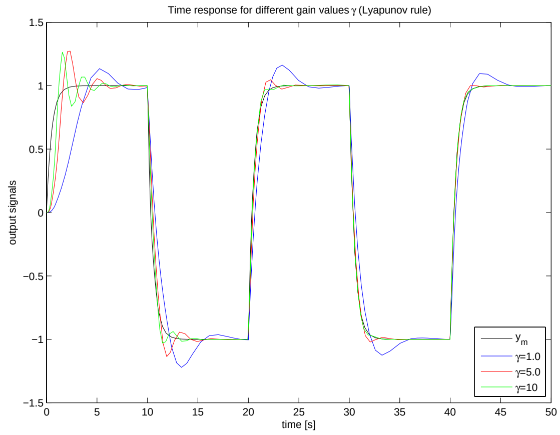

Fig. 5. Lyapunov rule: Time response for varying adaptation gain values γ.

TABLE II
LYAPUNOV RULE: SETTLING TIME AND OVERSHOOT OF FIRST STEP INPUT FOR VARYING ADAPTATION GAINS (APPROXIMATIONS FROM FIG. 5).

	overshoot (1. step)	peak time (1. step)	5% settling time (1. step)
y_m (reference)	-	-	1.48 s
$\gamma = 1.0$	13.5%	5.32 s	6.67 s
$\gamma = 5.0$	28.8%	2.21 s	5.30 s
$\gamma = 10$	27.0%	1.61 s	3.90 s

using the MIT rule, the controller eventually adapts to the true values and the estimation error converges to zero. The most significant difference in these results is that no instability can be observed. Even when setting $\gamma = 100$, convergence is maintained though with increased overshoot. Using the MIT rule on the other hand yields unstable behaviour for these high gain values.

C. Influence of changing reference model parameters

Lastly, the influence of changing the reference model's parameters a_m and b_m is examined. As before, the plant process parameters are set to $a = 1.0$ and $b = 0.5$. The adaptation gain is now chosen as $\gamma = 1.0$.

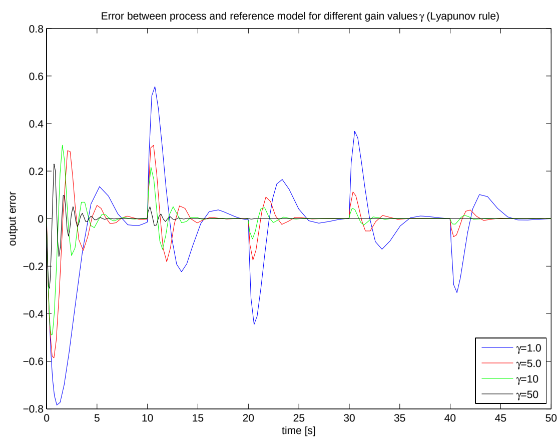

Fig. 6. Lyapunov rule: Tracking error for varying adaptation gain values γ.

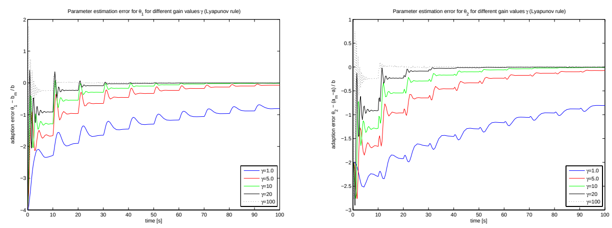

Fig. 7. Lyapunov rule: Parameter estimation error for θ_1 (left) and θ_2 (right) for varying adaptation gain values γ.

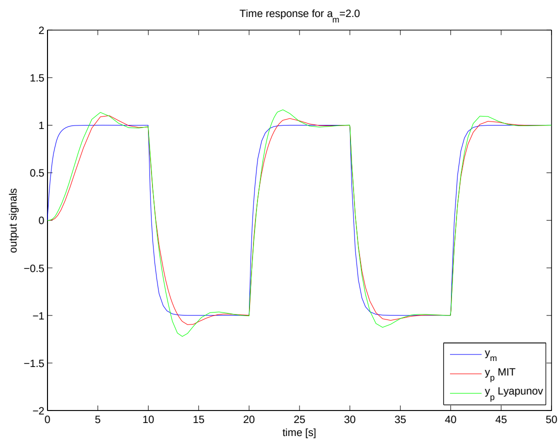

Fig. 8. Time response for $a_m = 2.0$.

The first parameter a_m is the time constant of the process and higher values result in the reference system's step response approaching the end value faster. Changing the reference model is equivalent to requesting a different time response for the plant process. The objective of this experiment is therefore to investigate whether the adaptive controller will successfully speed up the plant's response as well.

Fig. 8 shows the time responses of plant process y_p and reference model y_m for the default value $a_m = 2.0$. Only a slight difference between MIT rule and Lyapunov rule can be noticed with the MIT rule showing less overshoot.

Increasing the time constant to $a_m = 10$ results in the expected shorter rise time for y_m as seen in fig. 9. For both update laws the plot of y_p shows a reduced rise time as well, but significant overshoot prevents it from reaching the desired behaviour of shorter settling time. Therefore, the adaptation mechanism itself is not fast enough for adapting to very small time constant. Almost no difference can be seen between MIT and Lyapunov adaptation strategies.

On the other hand, reducing the parameter value to $a_m = 0.5$ and $a_m = 0.3$ and thus increasing the time constant results in the behaviour which is depicted in fig. 10 and 11. In these two cases, the MIT rule (red) does not yield successful adaptation whereas using the Lyapunov rule (green) shows excellent results and almost perfect following of the reference trajectory (blue).

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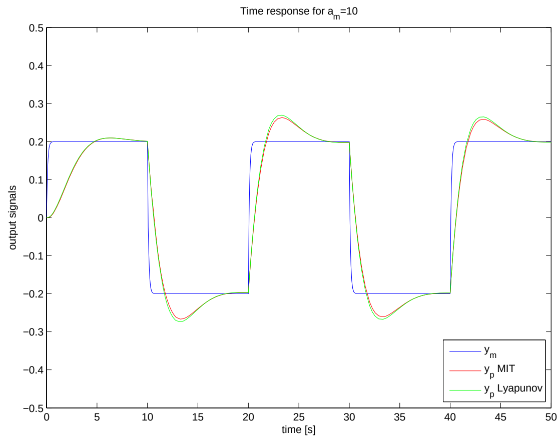

Fig. 9. Time response for $a_m = 10$.

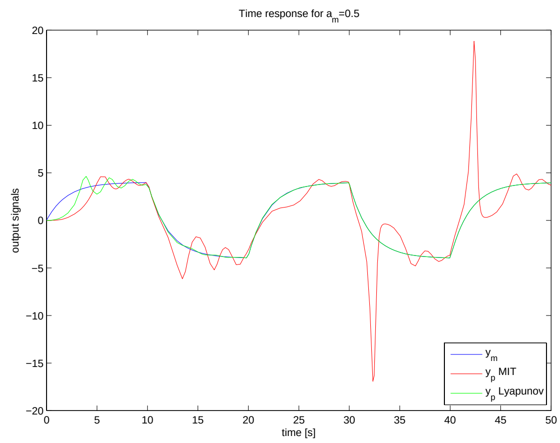

Fig. 10. Time response for $a_m = 0.5$.

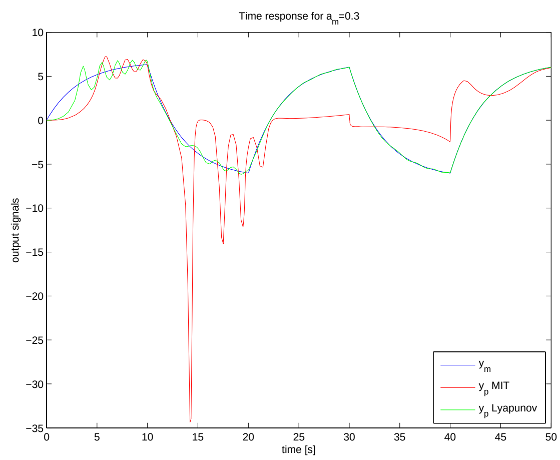

Fig. 11. Time response for $a_m = 0.3$.

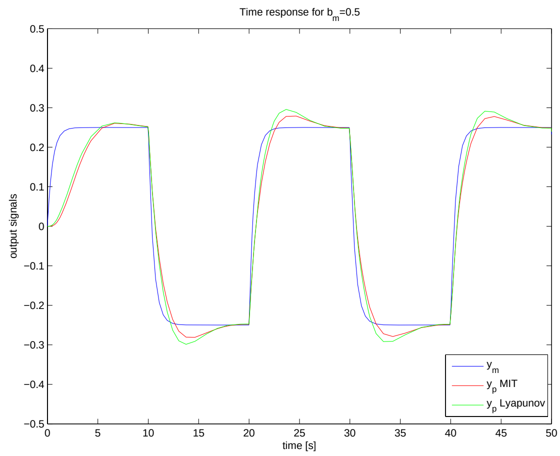

Fig. 12. Time response for $b_m = 0.5$.

Further experiments with different value combinations reveal that significant differences between MIT and Lyapunov rule occur if a_m is considerably lower than a. On the other hand, in the case of $a_m > a$ the results are almost similar. Taking into account the similarities between the two different update laws in eq. (25) and (30) as well as eq. (26) and (31) gives an idea of the underlying reason: The only difference is that the MIT rule features an additional dynamic term which resembles a low-pass structure. As this filter term incorporates a_m as its time constant, lower values than a therefore result in a delay which is greater than the delay of the process. This suggests that in the example discussed here the MIT rule update law is conceptually slower than required if $a_m < a$.

The second parameter b_m determines the system gain and the resulting output error is examined for different values of b_m (the plant process features $b = 0.5$). It can be seen from the results in fig. 12 to 15 that higher values for this *reference model gain* parameter cause similar behaviour as already seen when varying the *adaptation gain*.

Although the MIT rule may lead to instability for high gain values as seen in fig. 15 ($b_m = 50$), it also shows advantages in the case $b_m = 10$ (fig. 14). In contrast to the results when using the Lyapunov rule, the oscillations in the time response are of smaller amplitude. This can be related to the additional filter term in the MIT rule which damps oscillations of certain frequencies in the output signal. The high-frequency oscillations observed here are a well-known phenomenon which occurs when using high-gain control. As these pose a serious problem in practical applications, modified MRAC algorithms have been developed [8] and [9].

V. Conclusion

Model reference adaptive control is intended to ensure that a system with unknown or time-variant system parameters yields a desired closed-loop behaviour. The foundations of MRAC using two different adaptation laws are outlined in the first part of this article.

The second part describes the behaviour of a simulated first order system with two unknown parameters. It is analysed

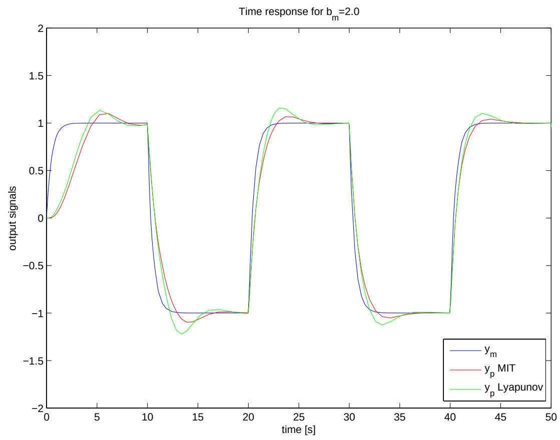

Fig. 13. Time response for $b_m = 2.0$.

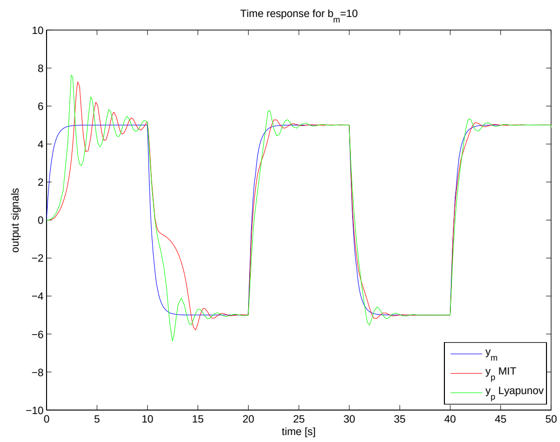

Fig. 14. Time response for $b_m = 10$.

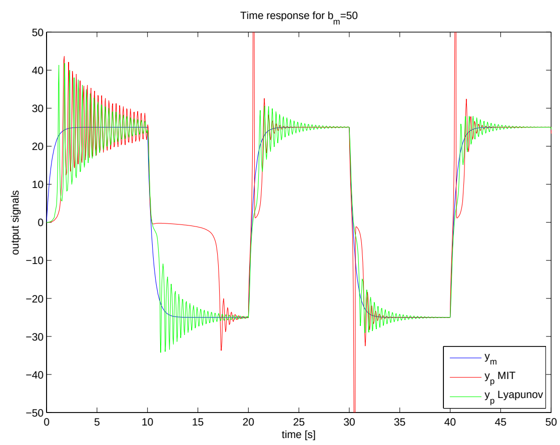

Fig. 15. Time response for $b_m = 50$.

how different values for the adaptation gain parameter affect the overall system performance. The results achieved in [1] and [2] are confirmed. The Lyapunov rule adaptation law is found to be superior to the MIT rule in these cases.

Furthermore, changes in the reference model parameters are analysed for both update laws. The effects which are observed here are then related to the different mathematical structures of the update laws. While both lead to similar results if $a_m > a$, the Lyapunov rule shows superior behaviour in the case $a_m < a$.

In [10] the effect of adaptation gain for a second order system using the MIT rule is analysed. Based on the results in this article, an interesting topic for further research would be to discuss second order systems using both MIT rule and Lyapunov rule and also focus on changes to reference model parameters.

It can be concluded that although an adaptive controller may control a plants behaviour to match the one of a predefined reference model in theory, several limitations underly the practical realisation. Rigorous testing and simulation with relevant input signals and parameter choices are necessary in order to ensure satisfactory behaviour.

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