Control Theory and Technology

On linear observers and application to fault detection in synchronous generators

Jan Erik Stellet [1†], Tobias Rogg [2]

1*Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany*

2*Swiss Federal Institute of Technology, 8092, Zurich, Switzerland*

Received 07 March 2013; revised 12 December 2013; revised 07 September 2014; accepted 08 September 2014

Abstract:

This work introduces an observer structure and highlights its distinct advantages in fault detection and isolation. Its application to the issue of shorted turns detection in synchronous generators is demonstrated. For the theoretical foundation, the convergence and design of Luenberger-type observers for disturbed LTI SISO systems are reviewed with a particular focus on input and output disturbances. As an additional result, a simple observer design for stationary output disturbances that avoids a system order extension, as in classical results, is proposed.

Keywords: synchronous generators, field winding, fault detection, unknown input observer, disturbance observer, residual generation

---

1 Introduction

Initially introduced in 1964 by Luenberger [1], state observers for linear time invariant systems form an integral part of state space control. The following advances termed Reduced Order Observers (ROO) [2] consider the separation of the state space into a measurable and an immeasurable subspace. Designing observers with minimal or partially reduced order has been studied [3].

Special emphasis is put on fault-tolerant observers. Considerable work has been devoted to the design of Unknown Input Observers (UIO) [4–9] which converge despite the presence of disturbances in the system equation [10]. Modelling sensor errors in an extended system state [11], both uncertainties in system and measurement equation can be represented as unknown input signals.

More recent approaches include adaptive control techniques for observer design [12] but are limited to constant or slowly time-varying disturbances. High-gain observers [13] can be used to reduce the influence of disturbances to an arbitrarily small level. However, this approach suffers from the amplification of measurement and process noise. Employing an extended descriptor system, this limitation can be alleviated [14]. Moreover, dynamic observers [15] are suitable for fault-tolerant observation without increasing the dimension of system equations [16]. Focusing on the effect of a disturbance on a control system input rather than on the disturbed states is considered by equivalent-input-disturbance estimators [17].

In this work, several relations between Luenberger-type observers in the presence of disturbances are studied from a theoretical point of view. This contribution explicitly details the general results obtained in [4, 6] concerning the relationship between unknown input and reduced order observers in the SISO case. Furthermore, conditions and simplified observer design methods for systems with disturbances in input and measurement are analysed.

Increasing attention is paid to the application of state observers in model based fault detection and isolation (FDI) [18–21]. The basic idea is to utilise the guaranteed convergence of an observer in the fault-free case to detect deviations in the system plant [22]. Henceforth, the output estimation error (residual) is monitored. Recently, the simple yet comprehensive notion of Total Measurable Fault Information Residual (ToMFIR) has been studied [23].

A key challenge is that the output residual usually also depends on quantities other than the fault's magnitude itself. This effect has to be compensated for in threshold-based detection schemes, which poses an additional problem if uncertain or time-varying parameters are involved.

†Corresponding author. Email: jan.stellet@student.kit.edu.

In previous results [24] gain-dependent scaling has been investigated for a Luenberger observer design which will be extended in this work to other observer types.

Moreover, a detailed analysis is presented for shorted turns detection in field windings of synchronous generators. Failures of the winding insulation are frequent, difficult to detect [25] and can lead to severe generator damage [26]. They have recently been studied in [27] for a machine with constant frequency. Here, special emphasis is put on variable frequency generators as used in wind turbines [28, 29] or naval and aircraft systems [30–32].

This work is organised as follows: section 2 constitutes definitions, background and furthermore derives an explicit formula for a Reduced Order Observer for SISO systems. In section 3, observer convergence and design in disturbed systems is analysed. Additionally, the close relationship between the ROO and the UIO is highlighted. In section 4, threshold-based fault detection is studied in general and for the application of shorted-turns detection in synchronous generators. All findings are summarised in section 5.

2 Background and definitions

2.1 Full-state Luenberger state space observer

A linear system is fully characterised by $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, $\boldsymbol{b} \in \mathbb{R}^n$ and $\boldsymbol{c} \in \mathbb{R}^n$ in its state space representation with the state vector $\boldsymbol{x}(t) \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}$ and the output $y(t) \in \mathbb{R}$:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \tag{1a}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t) \ . \tag{1b}$$

Only observable systems with regular observability matrix $\boldsymbol{Q}_\mathrm{B}$ are considered in this work. The well-known identity observer as proposed by Luenberger [1] is given as:

$$\dot{\hat{\boldsymbol{x}}}(t) = (\boldsymbol{A} - \boldsymbol{l}_\mathrm{L}\boldsymbol{c})\,\hat{\boldsymbol{x}}(t) + \boldsymbol{b}u(t) + \boldsymbol{l}_\mathrm{L}y(t) \ . \tag{2}$$

The dynamics of the full-state observer in (2) are determined by the matrix $\boldsymbol{N}_\mathrm{L} := \boldsymbol{A} - \boldsymbol{l}_\mathrm{L}\boldsymbol{c}$. The observer gain $\boldsymbol{l}_\mathrm{L}$ can be obtained using Ackermann's formula [33]

$$\boldsymbol{l}_\mathrm{L} = \left(f_0\boldsymbol{I}_n + f_1\boldsymbol{A} + \cdots + f_{n-1}\boldsymbol{A}^{n-1} + \boldsymbol{A}^n\right)\boldsymbol{s}_1 \tag{3}$$

with $f_0, \ldots, f_{n-1}$ being the coefficients of the desired characteristic polynomial. With $\boldsymbol{e}_n$ denoting the n-th canonical unit vector, $\boldsymbol{s}_1$ is defined as the last column of the inverted observability matrix:

$$\boldsymbol{s}_1 = \boldsymbol{Q}_\mathrm{B}^{-1}\boldsymbol{e}_n \ . \tag{4}$$

2.2 Reduced Order Observer

Many practical systems posses states that are metrologically accessible and do not need to be estimated. The idea of a Reduced Order Observer (ROO) as opposed to the full-state observer is to derive an estimate in the immeasurable state subspace only [2].

For SISO systems (1), consider the special case where the output $y(t)$ would be identical to a particular state $x_i(t)$. In this case, the measurable subspace is orthogonal to the immeasurable part $\boldsymbol{r}(t) \in \mathbb{R}^{n-1}$. It stands to reason to reorder and split up (1a) to obtain:

$$\begin{bmatrix} \dot{\boldsymbol{r}}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{r}(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{b}_1 \\ b_2 \end{bmatrix} u(t) \ . \tag{5}$$

Considering the last row of (5) as a measurement equation for the system determined by the first $n-1$ equations, an identity observer for $\boldsymbol{r}(t)$ can be derived according to (2). This yields the ROO formula [2]:

$$\begin{aligned} \dot{\boldsymbol{\eta}}(t) = {} & (\boldsymbol{A}_{11} - \boldsymbol{f}\boldsymbol{A}_{21})\,\boldsymbol{\eta}(t) + (\boldsymbol{b}_1 - \boldsymbol{f}b_2)\,u(t) && \text{(6a)} \\ & + [(\boldsymbol{A}_{11} - \boldsymbol{f}\boldsymbol{A}_{21})\,\boldsymbol{f} + \boldsymbol{A}_{12} - \boldsymbol{f}A_{22}]\,y(t) \\ \hat{\boldsymbol{r}}(t) = {} & \boldsymbol{\eta}(t) + \boldsymbol{f}y(t) \ . && \text{(6b)} \end{aligned}$$

The observer's dynamic is given by the eigenvalues of $(\boldsymbol{A}_{11} - \boldsymbol{f}\boldsymbol{A}_{21})$. Therefore, the gain vector $\boldsymbol{f}$ can be chosen by pole placement for this expression. By supplementing $\hat{\boldsymbol{r}}(t)$ with the measurement $y(t)$, the complete estimate $\hat{\boldsymbol{x}}(t)$ is obtained.

In general, the measurable subspace is not strictly orthogonal to the immeasurable subspace. However, any observable SISO system (1) can be transformed to its observable canonical form using the transformation:

$$\boldsymbol{z}(t) = \boldsymbol{T}^{-1}\boldsymbol{x}(t) \tag{7}$$

where $\boldsymbol{T}$ is given with $\boldsymbol{s}_1$ from (4) as [33]:

$$\boldsymbol{T} = \left[\boldsymbol{s}_1 \ \boldsymbol{A}\boldsymbol{s}_1 \ \ldots \ \boldsymbol{A}^{n-1}\boldsymbol{s}_1\right] \ . \tag{8}$$

In the transformed system, only the n-th element $z_n(t)$ of the transformed state vector spans the one-dimensional measurable subspace. Therefore, orthogonality to the immeasurable subspace is achieved.

2.3 Reduced Order Observer explicit form

This section details a constructive derivation of an n-th order observer formula different to the full-state observer (2). Considering a transformation (7) of system (1) to canonical form, the Reduced Order Observer formula (6) is applied. The main result is an explicit form of the reduced state space observer which will give further insight when compared to other observer types.

Lemma 1 (Reduced Order Observer explicit form) A state observer for a system (1) that is derived on the basis of (6) is given by

$$\dot{\boldsymbol{\rho}}(t) = \boldsymbol{N}_{\mathrm{R}}\boldsymbol{\rho}(t) + \boldsymbol{g}_{\mathrm{R}}u(t) + \boldsymbol{h}_{\mathrm{R}}y(t) \tag{9a}$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{\rho}(t) + \boldsymbol{l}_{\mathrm{R}}y(t) \tag{9b}$$

with

$$\boldsymbol{N}_{\mathrm{R}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{R}}\boldsymbol{c})\,\boldsymbol{A} \tag{10a}$$

$$\boldsymbol{g}_{\mathrm{R}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{R}}\boldsymbol{c})\,\boldsymbol{b} \tag{10b}$$

$$\boldsymbol{h}_{\mathrm{R}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{R}}\boldsymbol{c})\,\boldsymbol{A}\boldsymbol{l}_{\mathrm{R}} \tag{10c}$$

$$\boldsymbol{l}_{\mathrm{R}} = \left(f_0\boldsymbol{I}_n + \cdots + f_{n-2}\boldsymbol{A}^{n-2} + \boldsymbol{A}^{n-1}\right)\boldsymbol{s}_1 \tag{10d}$$

and with the initial value $\boldsymbol{\rho}(0)$ chosen in order to satisfy:

$$\boldsymbol{c}\boldsymbol{\rho}(0) = \boldsymbol{0}\ . \tag{11}$$

Proof See Appendix A.

With the system (1a) and by differentiating (9b), the observer error can be set forth:

$$\begin{aligned}\dot{\boldsymbol{e}}(t) &= \dot{\boldsymbol{x}}(t) - \dot{\hat{\boldsymbol{x}}}(t)\\ &= (\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t)) - (\dot{\boldsymbol{\rho}}(t) + \boldsymbol{l}_{\mathrm{R}}\dot{y}(t))\\ &= \boldsymbol{N}_{\mathrm{R}}\boldsymbol{e}(t)\ .\end{aligned} \tag{12}$$

Obviously, the observer error converges to zero if $\boldsymbol{l}_{\mathrm{R}}$ is chosen in order to constitute a stable system matrix. The coefficients $f_0, \ldots, f_{n-2}$ of its characteristic polynomial are found in (10d). The n-th eigenvalue of the system matrix $\boldsymbol{N}_{\mathrm{R}}$ equals zero:

$$0 = s\left(f_0 + f_1 s + \cdots + f_{n-2}s^{n-2} + s^{n-1}\right)\ . \tag{13}$$

The resulting observer formula (9) constitutes an equivalence to the general ROO (6). Upon this, new theoretical insight will be established in the following sections.

3 Observers for systems with disturbances

In the following, disturbances on the ideal system (1) are taken into consideration. Commonly experienced causes for such deviations are parameter uncertainties, sensor errors or unmodelled system behaviour. Here, a disturbed system is modelled as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{d}v(t) \tag{14a}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t) + w(t) \tag{14b}$$

with $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{b} \in \mathbb{R}^n$, $\boldsymbol{c} \in \mathbb{R}^n$, the state vector $\boldsymbol{x}(t) \in \mathbb{R}^n$, the known input $u(t) \in \mathbb{R}$ and the output $y(t) \in \mathbb{R}$. There are two undesired disturbances in the shape of an unknown input $v(t) \in \mathbb{R}$ and an additive $w(t) \in \mathbb{R}$ output disturbance.

Note that there is a difference between the deterministic but unknown disturbances assumed in this work and disturbances in the form of stochastic processes. In the latter case, the popular Kalman Filter [34] yields optimal state estimates under the assumption of white Gaussian noise.

In the following section, necessary conditions for the design of an n-th order disturbance observer will be studied. After showing that it is not feasible to achieve resilience to both input and output disturbances of arbitrary nature, observer design for the two cases will be studied.

3.1 Conditions for disturbance observer design

In order to address disturbances $v(t)$ in the system equation (14a), Unknown Input Observers (UIO) have been developed. Here, necessary conditions for the design of a UIO will be reviewed in the presence of additional output disturbances $w(t)$ in measurement equation (14b).

Theorem 1 (Disturbance observer distinction) Convergence of a linear Luenberger-type observer of the form

$$\dot{\boldsymbol{\rho}}(t) = \boldsymbol{N}\boldsymbol{\rho}(t) + \boldsymbol{g}u(t) + \boldsymbol{h}y(t) \tag{15a}$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{\rho}(t) + \boldsymbol{l}y(t) \tag{15b}$$

for a system disturbed according to (14) is restricted to the case of either $v(t) \neq 0$ or $w(t) \neq 0$.

Proof Starting from the observer structure (15) where $\hat{\boldsymbol{x}}(t) \in \mathbb{R}^n$ is the state estimate, $\boldsymbol{\rho}(t) \in \mathbb{R}^n$. $\boldsymbol{N} \in \mathbb{R}^{n\times n}$, $\boldsymbol{l} \in \mathbb{R}^n$, $\boldsymbol{g} \in \mathbb{R}^n$ and $\boldsymbol{h} \in \mathbb{R}^n$ are matrices to be determined. The state estimation error for system (14) is:

$$\begin{aligned}\dot{\boldsymbol{e}}(t) = {}&(\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}\boldsymbol{A} + \boldsymbol{N}\boldsymbol{l}\boldsymbol{c} - \boldsymbol{h}\boldsymbol{c})\boldsymbol{x}(t) - \boldsymbol{N}\hat{\boldsymbol{x}}(t)\\ &+ (\boldsymbol{N}\boldsymbol{l} - \boldsymbol{h})w(t) - \boldsymbol{l}\dot{w}(t) + (\boldsymbol{b} - \boldsymbol{l}\boldsymbol{c}\boldsymbol{b} - \boldsymbol{g})u(t)\\ &+ (\boldsymbol{d} - \boldsymbol{l}\boldsymbol{c}\boldsymbol{d})v(t)\ .\end{aligned} \tag{16}$$

With $\boldsymbol{P} := (\boldsymbol{I}_n - \boldsymbol{l}\boldsymbol{c})$ and the following conditions

$$\boldsymbol{g} = \boldsymbol{P}\boldsymbol{b} \tag{17}$$

$$\boldsymbol{N}\boldsymbol{P} = \boldsymbol{P}\boldsymbol{A} - \boldsymbol{h}\boldsymbol{c} \tag{18}$$

(16) is simplified to become

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{N}\boldsymbol{e}(t) + (\boldsymbol{N}\boldsymbol{l} - \boldsymbol{h})w(t) - \boldsymbol{l}\dot{w}(t) + \boldsymbol{P}\boldsymbol{d}v(t)\ . \tag{19}$$

In order for the observer error $\boldsymbol{e}(t)$ to decay it is required that $\boldsymbol{N}$ constitutes a stable system dynamic with eigenvalues in the left half-plane. A necessary condition is thus that $\boldsymbol{N}$ is regular. The proof is completed within the following two lemmata where it is shown that it is not possible to find a regular $\boldsymbol{N}$ which yields independence of (19) against unknown inputs $v(t)$ and arbitrary output disturbances $w(t)$ at the same time.

Lemma 2 (Unknown Input Observer) An Unknown Input Observer requires $\boldsymbol{P}$ to be singular. This requires an additive matrix to give a regular system matrix $\boldsymbol{N}$:

$$\boldsymbol{N} = \boldsymbol{PA} + \boldsymbol{\Delta} \, . \tag{20}$$

Proof Making (19) independent of the unknown input $v(t)$ requires:

$$\boldsymbol{Pd} = \boldsymbol{0} \, . \tag{21}$$

For $\boldsymbol{d} \neq \boldsymbol{0}$, this holds true if and only if $\boldsymbol{P}$ is singular.

Lemma 3 (Output Disturbance Observer) Observer convergence in spite of an output disturbance requires $\boldsymbol{P}$ to be regular.

Proof Independence of $w(t)$ in (19) would require

$$\boldsymbol{h} = \boldsymbol{Nl} \tag{22}$$

which in turn determines $\boldsymbol{N}$ according to (18) to

$$\boldsymbol{N} = \boldsymbol{PA} \, . \tag{23}$$

With $\boldsymbol{A}$ assumed to be regular, regularity of $\boldsymbol{N}$ requires that $\boldsymbol{P}$ is regular.

Note that this gives only a necessary condition, as the disturbance's derivative $\dot{w}(t)$ was not considered. Obviously, there is no simple means for freeing (19) of an arbitrary disturbance at all times. In [7, 35] the special case of $w(t)$ being a linear combination of $v(t)$ is considered. However, in the relevant stationary case with $\dot{w}(t) = 0$, a necessary and sufficient condition is provided by (23).

3.2 Unknown Input Observer design

Considering the disturbed system (14) with $v(t) \neq 0$ and $w(t) = 0$ an approach to design an Unknown Input Observer will be studied. Compared with the derivation of the explicit Reduced Order Observer formula (9), a novel simple scheme is identified at the cost of only a minor restriction on the observer's initial value selection. Given the requirement (21), a way to choose $\boldsymbol{N}$ is presented in [5] and will be briefly reviewed for the SISO case. An alternative design method by a projection operator approach is presented in [36].

It is required that $(\boldsymbol{cd})^{-1}$ exists. Then, (21) determines

$$\boldsymbol{l}_{\mathrm{U}} = \boldsymbol{d}(\boldsymbol{cd})^{-1} \tag{24}$$

and therefore

$$\boldsymbol{P} = \boldsymbol{I}_n - \boldsymbol{d}(\boldsymbol{cd})^{-1}\boldsymbol{c} \, . \tag{25}$$

Postmultiplying (18) by $\boldsymbol{d}$ gives $\boldsymbol{h}_{\mathrm{U}}$:

$$\boldsymbol{h}_{\mathrm{U}} = \boldsymbol{PAl}_{\mathrm{U}} \, . \tag{26}$$

Considering the choice of $\boldsymbol{l}_{\mathrm{U}}$ and $\boldsymbol{h}_{\mathrm{U}}$ (18) becomes:

$$\boldsymbol{N}_{\mathrm{U}}\boldsymbol{P} = \boldsymbol{PAP} \, . \tag{27}$$

The general solution for $\boldsymbol{N}_{\mathrm{U}}$ in (27) is given by [5]:

$$\boldsymbol{N}_{\mathrm{U}} = \boldsymbol{PA} + \boldsymbol{\beta c} \, . \tag{28}$$

The approach pursued in (17), (18) and (21) is entirely different to the formulation of the ROO for an undisturbed system in section 2.3. However, the result in (28) is identical to $\boldsymbol{N}_{\mathrm{R}}$ in (10a) except for the additive term $\boldsymbol{\beta c}$.

Corollary 1 (Equivalence to Reduced Order Observer) Any UIO is a special case of the derived explicit ROO structure where $\boldsymbol{f}$ is not a free parameter but determined by $\boldsymbol{d}$ which characterises the unknown input.

Proof Equation (24) gives:

$$\boldsymbol{l}_{\mathrm{U}}^{*} = \boldsymbol{T}^{-1}\boldsymbol{d}\,(\boldsymbol{cd})^{-1} = \frac{\boldsymbol{T}^{-1}\boldsymbol{d}}{(\boldsymbol{cT})\left(\boldsymbol{T}^{-1}\boldsymbol{d}\right)} = \begin{bmatrix} \boldsymbol{f} \\ 1 \end{bmatrix} \, . \tag{29}$$

Thus, $\boldsymbol{l}_{\mathrm{U}}^{*}$ has the same form as $\boldsymbol{l}_{\mathrm{R}}^{*}$ in (10d). Equivalence of $\boldsymbol{N}_{\mathrm{U}}$, $\boldsymbol{g}_{\mathrm{U}}$ and $\boldsymbol{h}_{\mathrm{U}}$ to the ROO design (10) then follows from (28), (17) and (26).

Note that the UIO only converges if $\boldsymbol{f}$ constitutes a stable polynomial (13). Additionally, $\boldsymbol{\beta}$ which determines the n-th eigenvalue needs to be chosen accordingly to ensure convergence. As the observer eigenvalues can only be partially assigned, the system is not fully observable. This becomes obvious when calculating $\boldsymbol{Q}_{\mathrm{B}}$ from $(\boldsymbol{PA},\, \boldsymbol{c})$ [8].

An interesting consequence of this result is that despite the complex procedure to determine $\boldsymbol{N}_{\mathrm{U}}$ for the UIO in (28), a simple form is obtained if a relatively mild constraint on the initial value $\boldsymbol{\rho}(0)$ is imposed.

Corollary 2 (Unknown Input Observer Simplification) If the initial value is restricted in order for

$$\boldsymbol{c\rho}(0) = 0 \tag{30}$$

to hold, the system matrix (28) is reduced to $\boldsymbol{N}_{\mathrm{U}} = \boldsymbol{PA}$.

Proof The proof is given by the constructive derivation of the ROO in section 2.3. Here, condition (11) is imposed in order to achieve the state extension.

3.3 Output Disturbance Observer design

For completeness, observer design in the presence of additive output disturbances is studied.

Consider system (14) with $v(t) = 0$ and $w(t) \neq 0$. As has been pointed out, making the state estimation error (19) independent of $w(t)$ requires additional knowledge on the disturbance. One approach assumes a model of the disturbance's dynamics and extends the system state [11]. The enhanced system state comprises $w(t)$ as a combination of k additional states $\boldsymbol{x}_w(t)$ which are represented by a linear dynamic:

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_w(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_w \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}_w(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{0} \end{bmatrix} u(t) \tag{31a}$$

$$y(t) = \begin{bmatrix} \boldsymbol{c}\, \boldsymbol{c}_w \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}_w(t) \end{bmatrix} . \tag{31b}$$

Observer design for the enhanced system state can be performed using a full-state observer (3) of order $n + k$.

If however, no information on the disturbance is given except that it exhibits stable dynamics, this approach reduces to the stationary state of the disturbance:

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{b} \\ 0 \end{bmatrix} u(t) \tag{32a}$$

$$y(t) = \begin{bmatrix} \boldsymbol{c}\; 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ w(t) \end{bmatrix} . \tag{32b}$$

Note that another way to represent arbitrary output disturbances is to enhance the above system with an unknown input signal in the $(n + 1)$-th component. However, [5] established that in this case, observer design is restricted to systems with stable $\boldsymbol{A}$ and is therefore not practical.

Lemma 4 (System Matrix and Observability) Assuming that the undisturbed system (1) is observable, the extended system (32) is observable if and only if $\boldsymbol{A}$ is regular.

Proof See Appendix B.

Studying observer design for this system of order $n + 1$ using the reduced order formula from section 2.3 will be presented in the following.

Theorem 2 (Stationary Output Disturbance Observer) Let a system (14) with non-singular $\boldsymbol{A}$, $v(t) = 0$ and $w(t) \neq 0$ be given. Then, the following generalisation of (9) constitutes an n-th order state observer for the system with stationary output disturbance:

$$\dot{\boldsymbol{\rho}}(t) = \boldsymbol{N}_{\mathrm{G}}\boldsymbol{\rho}(t) + \boldsymbol{g}_{\mathrm{G}}u(t) + \boldsymbol{h}_{\mathrm{G}}y(t) \tag{33a}$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{\rho}(t) + \boldsymbol{l}_{\mathrm{G}}y(t) \tag{33b}$$

with

$$\boldsymbol{N}_{\mathrm{G}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\,\boldsymbol{A} \tag{34a}$$

$$\boldsymbol{g}_{\mathrm{G}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\,\boldsymbol{b} \tag{34b}$$

$$\boldsymbol{h}_{\mathrm{G}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\,\boldsymbol{A}\boldsymbol{l}_{\mathrm{G}} . \tag{34c}$$

Proof See Appendix C.

The observer structure (33), hereafter referred to as *Stationary Output Disturbance Observer*, is identical to the ROO with the only difference being a generalised gain vector $\boldsymbol{l}_{\mathrm{G}}$. Hence, the error dynamics (12) apply as well. Note that there is no condition on the initial value $\boldsymbol{\rho}(0)$.

In the following, the calculation of the observer gain is explored and related to the pole placement for a full-state Luenberger observer.

Lemma 5 (Observer Gain) The observer gain $\boldsymbol{l}_{\mathrm{G}}$ can be calculated with $\boldsymbol{s}_1$ from (4) and the coefficients of the desired characteristic polynomial $f_0, \ldots, f_{n-1}$ as:

$$\boldsymbol{l}_{\mathrm{G}} = \boldsymbol{A}^{-1}\left(f_0\boldsymbol{I}_n + \cdots + f_{n-1}\boldsymbol{A}^{n-1} + \boldsymbol{A}^n\right)\boldsymbol{s}_1 \tag{35}$$

$$= \boldsymbol{A}^{-1}\boldsymbol{l}_{\mathrm{L}} . \tag{36}$$

Here, $\boldsymbol{l}_{\mathrm{L}}$ denotes the gain vector of a full-state Luenberger observer (3) with the same poles.

Proof First, $\boldsymbol{l}_{\mathrm{R}}$ from (10d) is calculated for the $(n+1)$-dimensional system (32). Only the first n entries are considered which yields:

$$\boldsymbol{l}_{\mathrm{G}} = \left(f_0\boldsymbol{I}_n + \cdots + f_{n-1}\boldsymbol{A}^{n-1} + \boldsymbol{A}^n\right)\boldsymbol{s}_{\mathrm{G}} . \tag{37}$$

Here, $\boldsymbol{s}_{\mathrm{G}}$ denotes the first n components of $\boldsymbol{s} = \begin{bmatrix}\boldsymbol{s}_{\mathrm{G}}\; s_2\end{bmatrix}^\top$ where $\boldsymbol{s}$ is defined as in (4) but for the extended system (32). Premultiplying the definition of $\boldsymbol{s}$ with $\boldsymbol{Q}_{\mathrm{B}}$ gives:

$$\begin{bmatrix}\boldsymbol{c}\boldsymbol{s}_{\mathrm{G}} + s_2\; \boldsymbol{c}\boldsymbol{A}\boldsymbol{s}_{\mathrm{G}} \ldots \boldsymbol{c}\boldsymbol{A}^n\boldsymbol{s}_{\mathrm{G}}\end{bmatrix} = \boldsymbol{e}_n^\top . \tag{38}$$

The first of these $n + 1$ linear equations determines s_2. Comparing the following n equations with the definition of $\boldsymbol{s}_1$ for the undisturbed system in (4) gives that $\boldsymbol{s}_{\mathrm{G}} = \boldsymbol{A}^{-1}\boldsymbol{s}_1$. Inserting this relation in (37) gives the result (36).

Furthermore, the eigenvalues of $\boldsymbol{N}_{\mathrm{G}}$ achieved by $\boldsymbol{l}_{\mathrm{G}} = \boldsymbol{A}^{-1}\boldsymbol{l}_{\mathrm{L}}$ are identical to the eigenvalues of the system matrix $\boldsymbol{N}_{\mathrm{L}}$ of a full-state Luenberger observer with gain $\boldsymbol{l}_{\mathrm{L}}$:

$$\boldsymbol{N}_{\mathrm{G}} = (\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\boldsymbol{A} = (\boldsymbol{I}_n - \boldsymbol{A}^{-1}\boldsymbol{l}_{\mathrm{L}}\boldsymbol{c})\boldsymbol{A}$$

$$= \boldsymbol{A}^{-1}(\boldsymbol{A} - \boldsymbol{l}_{\mathrm{L}}\boldsymbol{c})\boldsymbol{A} = \boldsymbol{A}^{-1}\boldsymbol{N}_{\mathrm{L}}\boldsymbol{A} \ . \quad (39)$$

Matrix $\boldsymbol{N}_{\mathrm{L}}$ is similar to $\boldsymbol{N}_{\mathrm{G}}$ with similarity transformation $\boldsymbol{A}$. Thus, their eigenvalues are the same.

When compared to a full-state observer design, the reduced order of the observer (33) might give improvements in terms of computational requirements. Another advantage arises in fault diagnosis applications and will be explored in section 4.

On the other hand, a drawback shared by all observers in the form of (15) is the immediate dependence on the measurement $y(t)$. In contrast, a Luenberger observer type (2) acts as a low-pass filter on the measurement signal which is beneficial in the presence of measurement noise.

3.4 Summary of main results

In the first part of this section, conditions for observer convergence in the presence of unknown input as well as output disturbances are analysed. It is found that independence against both disturbances cannot be achieved with a single observer of order n.

Secondly, design of an Unknown Input Observer is reviewed. It is found that the UIO is a special case of the ROO. From this result, a simplification to the UIO system matrix is proposed which gives an easier path towards finding an observer at the cost of only a minor restriction on how the initial observer state is to be chosen.

Completing the study, the third section details observer design in the presence of output disturbances. As a generalisation of the explicit ROO formula from section 2.3, the Stationary Output Disturbance Observer is presented.

4 Observers for fault diagnosis

4.1 Methodology

A state observer incorporates a model of a physical system to estimate state variables. Deviations in the physical system that are not reflected in the model result in a residual error in the state estimates. This residual can therefore be used as an indicator of defects and aging [37, 38].

Given a system (1a) faults are modelled as:

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \delta\boldsymbol{A})\,\boldsymbol{x}(t) + (\boldsymbol{b} + \delta\boldsymbol{b})\,u(t) \ . \quad (40)$$

The error caused by changes in $\boldsymbol{A}$ and $\boldsymbol{b}$ can be combined to form an unknown input $\boldsymbol{\epsilon}(t) := \delta\boldsymbol{A}\boldsymbol{x}(t) + \delta\boldsymbol{b}u(t)$:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{\epsilon}(t) \ . \quad (41)$$

The Total Measurable Fault Information Residual [23] is

$$r(t) := y(t) - \hat{y}(t) = \boldsymbol{c}\,(\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)) \quad (42)$$

which converges to zero in the case of no faults. Of particular interest is the stationary limit (hereinafter called ToMFIR) that is caused by faults with stationary end value:

$$r_\infty = \lim_{t\to\infty} r(t) \ . \quad (43)$$

The ToMFIR of a full-state Luenberger observer (2) depends on the observer gain $\boldsymbol{l}_{\mathrm{L}}$:

$$r_\infty = -\boldsymbol{c}\,(\boldsymbol{A} - \boldsymbol{l}_{\mathrm{L}}\boldsymbol{c})^{-1}\,\boldsymbol{\epsilon}_\infty \ . \quad (44)$$

Large gains create a small residual, while gains that place the eigenvalues of the observer near zero create a high residual value for the same fault. As a remedy, [24] proposes a multiplicative compensation.

The additional correction is avoided for observer designs which generate residuals that are independent of the gain:

Corollary 3 (Stationary Output Disturbance Observer) A Stationary Output Disturbance Observer (33) can be used to produce ToMFIR values that are gain-independent:

$$r_\infty = -\boldsymbol{c}\boldsymbol{A}^{-1}\boldsymbol{\epsilon}_\infty \ . \quad (45)$$

Proof The residual for an observer design (15) is:

$$\begin{aligned} r_\infty = \boldsymbol{c}\,\big[&-\left(\boldsymbol{P} + \boldsymbol{N}^{-1}\boldsymbol{h}\boldsymbol{c}\right)\boldsymbol{A}^{-1}\boldsymbol{\epsilon}_\infty \\ &+ \left(\boldsymbol{N}^{-1}\boldsymbol{g} - \boldsymbol{P}\boldsymbol{A}^{-1}\boldsymbol{b} - \boldsymbol{N}^{-1}\boldsymbol{h}\boldsymbol{c}\boldsymbol{A}^{-1}\boldsymbol{b}\right)u_\infty\big] \ . \end{aligned} \quad (46)$$

Given that the proposed observer design (33) fulfils (17), (18), (22) and (23), the residual is reduced to (45).

4.2 Application to shorted turns detection

In the following, the application of threshold based fault detection in synchronous machines is studied. In [27] a Luenberger observer is employed for diagnosis of shorted turns in the field windings. Here, these results are extended using a Stationary Output Disturbance Observer (33) for residual generation. Besides the independence on the observer gain that has been discovered in the previous section, emphasis is put on the residual's dependence on system parameters, especially the electrical frequency.

A model of a synchronous machine is given by [27]:

$$\begin{bmatrix}\dot{I}_d(t)\\ \dot{I}_q(t)\end{bmatrix} = \begin{bmatrix}-\frac{R_s}{L_d} & \frac{L_q\,\omega_r}{L_d}\\ -\frac{L_d\,\omega_r}{L_q} & -\frac{R_s}{L_q}\end{bmatrix}\begin{bmatrix}I_d(t)\\ I_q(t)\end{bmatrix}$$

$$
+\begin{bmatrix} -\frac{1}{L_d} & 0 & 0 \\ 0 & -\frac{1}{L_q} & \frac{2\,L_m\,N_f\,\omega_r}{3\,L_q\,N_s} \end{bmatrix} \begin{bmatrix} U_d\left(t\right) \\ U_q\left(t\right) \\ I_f\left(t\right) \end{bmatrix} . \tag{47}
$$

The system state is given by stator direct and quadrature currents $I_d\left(t\right)$ and $I_q\left(t\right)$. Direct and quadrature voltages $U_d\left(t\right)$ and $U_q\left(t\right)$ as well as exciter current $I_f\left(t\right)$ form the system input.

The presence of a field winding fault reduces the effective number of turns to $\overline{N}_f$ and thus creates the unknown input signal:

$$
\boldsymbol{\epsilon}\left(t\right) = \delta \boldsymbol{b} u\left(t\right) = \begin{bmatrix} 0 \\ \frac{2\,I_f(t)\,L_m\,\omega_r\,\left(\overline{N}_f - N_f\right)}{3\,L_q\,N_s} \end{bmatrix} . \tag{48}
$$

The synchronous generator is defined by parameters which are explained in table 1.

Table 1 Parameter description and values used in simulation example.

Parameter	Symbol	Value [39]
Direct inductance	L_d	8.79×10^{-5} H
Quadrature inductance	L_q	3.96×10^{-5} H
Magnetising inductance	L_m	6.26×10^{-5} H
Stator resistance	R_s	$7.79 \times 10^{-3}\,\Omega$
Electrical frequency	ω_r	$2\pi \times 10^3\,\mathrm{s}^{-1}$
Exciter current	I_f	154.93 A
No. of turns per stator phase	N_s	1
No. of turns of field winding	N_f	10
No. of turns after insulation fault	$\overline{N}_f$	9

A full-state Luenberger observer (2) can be employed to calculate residuals under the assumption that both system states $I_d\left(t\right), I_q\left(t\right)$ are measurable. As has been shown in [27], the stationary residual (44) is

$$
\boldsymbol{r}_\infty = \begin{bmatrix} 0 \\ \frac{I_f'\,L_m\,\omega_r}{L_q\,\lambda_2}\,\frac{\left(\overline{N}_f - N_f\right)}{N_f} \end{bmatrix} \tag{49}
$$

with λ_2 as the second observer eigenvalue and the constant stator-side referred exciter current $I_f' = 2 I_f N_f / 3 N_s$ [40].

In order to achieve independence of quantities other than the number of windings in (49), [27] proposes to set:

$$
\lambda_2 = -\frac{I_f'\,L_m\,\omega_r}{L_q} . \tag{50}
$$

However, this in turn limits the detection speed of the observer to a fixed value. Moreover, the compensation (50) requires setting λ_2 proportional to ω_r which may result in increased noise sensitivity in high frequency machines. Additional difficulties arise in variable frequency applications [28–32].

On the other hand, when using a SODO, the residual (45) is independent of the eigenvalue locations:

$$
\boldsymbol{r}_\infty = \begin{bmatrix} -\frac{I_f'\,L_m\,L_q\,\omega_r^2}{\left(R_s^2 + L_d\,L_q\,\omega_r^2\right)}\,\frac{\left(\overline{N}_f - N_f\right)}{N_f} \\ -\frac{I_f'\,L_m\,R_s\,\omega_r}{\left(R_s^2 + L_d\,L_q\,\omega_r^2\right)}\,\frac{\left(\overline{N}_f - N_f\right)}{N_f} \end{bmatrix} . \tag{51}
$$

Note that $\boldsymbol{A}$ in (47) is always regular, guaranteeing the existence of the observer.

Results of a simulation example visualise the advantage of having additional degrees of freedom in the observer design. Figure 1 shows that varying λ_2 in relation to the constant value (50) produces improved detection time.

Another significant advantage is that the first component $r_{\infty,1}$ in (51) allows for drastic simplification for typical configurations, leading to the following corollary.

Corollary 4 (SODO residual simplification) For typical synchronous generators [41,42] it holds that:

$$
\frac{R_s^2}{L_d\,L_q\,\omega_r^2} \ll 1 . \tag{52}
$$

Then, the first component of residual (51) is simplified to:

$$
r_{\infty,1} = -I_f' \frac{L_m}{L_d} \frac{\left(\overline{N}_f - N_f\right)}{N_f} . \tag{53}
$$

This residual directly gives the per cent of the field windings that have short circuited multiplied by the transformed stator current and L_m/L_d, a factor that is usually close to 1. It is therefore an ideal fault indicator, as minimal dependency on uncertain or varying parameters is achieved.

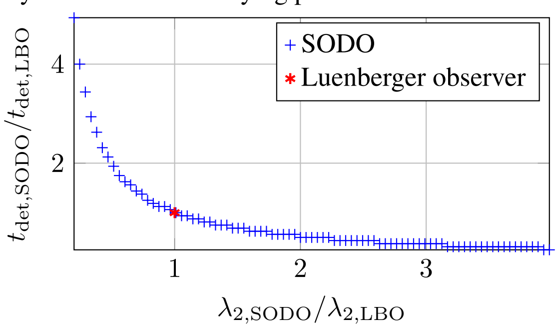

Fig. 1 Detection time for a threshold of 80% employing a Stationary Output Disturbance Observer (SODO) (33) compared to a conventional Luenberger observer (LBO) (2) for a simulation of

one faulted turn with the machine in [39]. Having a residual independent of the eigenvalues, the SODO can be designed with an arbitrary λ_2. This enables it to detect the fault in only a fraction of the time that the Luenberger observer would have needed.

5 Conclusion

This contribution constructs a unique observer structure for fault detection and isolation. This is achieved by developing a distinct explicit form of a Reduced Order Observer and establishing theoretical relations with other observer types. Secondly, design of linear observers for systems with stationary output disturbances is considered. In contrast to classical results, an extension of the system order is avoided while maintaining a particularly simple design procedure.

Based on this, recent results in the application of model based fault detection are extended. Compared to a Luenberger identity observer, it is found that gain-dependent scaling of the residual is avoided with the novel design.

Exemplifying the general result, further application specific advantages are found for shorted turns detection in synchronous generators. Here, recently obtained results are extended and an improved fault detection scheme is studied. The residual expressions of the proposed observer design stand out not only by the absence of undesired scaling, but exhibit further advantages in the form of minimal parameter dependence due to a simplification applicable to most generators in use today.

References

[1] D. G. Luenberger, "Observing the state of a linear system," *Military Electronics, IEEE Transactions on*, vol. 8, no. 2, pp. 74–80, 1964.

[2] D. Luenberger, "An introduction to observers," *Automatic Control, IEEE Transactions on*, vol. 16, no. 6, pp. 596–602, 1971.

[3] S. Balemi, "Partial-order reduction of observers for linear systems," in *Proceedings of the 17th IFAC World Congress*, Seoul, Korea, 2008, pp. 7723–7728.

[4] R. J. Miller and R. Mukundan, "On designing reduced-order observers for linear time-invariant systems subject to unknown inputs," *International Journal of Control*, vol. 35, no. 1, pp. 183–188, 1982.

[5] F. Yang and R. Wilde, "Observers for linear systems with unknown inputs," *Automatic Control, IEEE Transactions on*, vol. 33, no. 7, pp. 677–681, 1988.

[6] M. Hou and P. Muller, "Design of observers for linear systems with unknown inputs," *Automatic Control, IEEE Transactions on*, vol. 37, no. 6, pp. 871–875, 1992.

[7] ——, "Disturbance decoupled observer design: a unified viewpoint," *Automatic Control, IEEE Transactions on*, vol. 39, no. 6, pp. 1338–1341, 1994.

[8] M. Darouach, M. Zasadzinski, and S. Xu, "Full-order observers for linear systems with unknown inputs," *Automatic Control, IEEE Transactions on*, vol. 39, no. 3, pp. 606–609, 1994.

[9] M. Lungu and R. Lungu, "Full-order observer design for linear systems with unknown inputs," *International Journal of Control*, vol. 85, no. 10, pp. 1602–1615, 2012.

[10] A. Samantaray and B. Bouamama, *Model-based Process Supervision: A Bond Graph Approach*, ser. Advances in Industrial Control. Springer, 2008.

[11] J. Park, G. Rizzoni, and W. B. Ribbens, "On the representation of sensor faults in fault detection filters," *Automatica*, vol. 30, no. 11, pp. 1793–1795, 1994.

[12] H. Wang and S. Daley, "Actuator fault diagnosis: an adaptive observer-based technique," *Automatic Control, IEEE Transactions on*, vol. 41, no. 7, pp. 1073–1078, 1996.

[13] H. Hammouri, M. Kinnaert, and E.-H. El Yaagoubi, "Observer-based approach to fault detection and isolation for nonlinear systems," *Automatic Control, IEEE Transactions on*, vol. 44, no. 10, pp. 1879–1884, 1999.

[14] Z. Gao, T. Breikin, and H. Wang, "High-gain estimator and fault-tolerant design with application to a gas turbine dynamic system," *Control Systems Technology, IEEE Transactions on*, vol. 15, no. 4, pp. 740–753, 2007.

[15] J.-K. Park, D.-R. Shin, and T. M. Chung, "Dynamic observers for linear time-invariant systems," *Automatica*, vol. 38, no. 6, pp. 1083–1087, 2002.

[16] A. Wahrburg and J. Adamy, "Robust fault isolation using dynamically extended observers," in *Intelligent Control (ISIC), 2012 IEEE International Symposium on*, Dubrovnik, Croatia, 2012, pp. 1201–1206.

[17] M. Wu, K. Lou, F. Xiao, R. Liu, Y. He, and J. She, "Design of equivalent-input-disturbance estimator using a generalized state observer," *Journal of Control Theory and Applications*, vol. 11, no. 1, pp. 74–79, 2013.

[18] K. Emami, B. Nener, V. Sreeram, H. Trinh, and T. Fernando, "A fault detection technique for dynamical systems," in *Industrial and Information Systems (ICIIS), 8th IEEE International Conference on*, Peradeniya, Sri Lanka, 2013, pp. 201–206.

[19] I. Hwang, S. Kim, Y. Kim, and C. Seah, "A survey of fault detection, isolation, and reconfiguration methods," *Control Systems Technology, IEEE Transactions on*, vol. 18, no. 3, pp. 636–653, 2010.

[20] T. Sellami, H. Berriri, S. Jelassi, and M. Mimouni, "Sliding mode observer-based fault-detection of inter-turn short-circuit in induction motor," in *Sciences and Techniques of Automatic Control and Computer Engineering (STA), 14th International Conference on*, Sousse, Tunisia, 2013, pp. 524–529.

[21] S. Shao, P. Wheeler, J. Clare, and A. Watson, "Fault detection for modular multilevel converters based on sliding mode observer," *Power Electronics, IEEE Transactions on*, vol. 28, no. 11, pp. 4867–4872, Nov 2013.

[22] K. Zhang, B. Jiang, and P. Shi, *Observer-Based Fault Estimation and Accomodation for Dynamic Systems*, ser. Lecture Notes in Control and Information Sciences. Berlin Heidelberg: Springer, 2012, vol. 436.

[23] W. Chen, F. N. Chowdhury, A. Djuric, and C.-P. Yeh, “Robust fault detection of turbofan engines subject to adaptive controllers via a total measurable fault information residual (tomfir) technique,” *ISA Transactions*, 2014, in press.

[24] F. Chowdhury and W. Chen, “A modified approach to observer-based fault detection,” in *Intelligent Control, IEEE 22nd International Symposium on*, Singapore, Singapore, 2007, pp. 539–543.

[25] R. Fier, D. Makuc, H. Lavri, D. Miljavec, and M. Bugeza, “Modeling, analysis and detection of rotor field winding faults in synchronous generators,” in *Electrical Machines (ICEM), XIX International Conference on*, Rome, Italy, 2010.

[26] C. Gaona, F. Blazquez, P. Frias, and M. Redondo, “A novel rotor ground-fault-detection technique for synchronous machines with static excitation,” *Energy Conversion, IEEE Transactions on*, vol. 25, no. 4, pp. 965–973, 2010.

[27] T. Batzel, “Observer-based monitoring of synchronous generator winding health,” in *Power Systems Conference and Exposition, PSCE ’06 IEEE PES*, Atlanta, USA, 2006, pp. 1150–1155.

[28] A. Yazdani and R. Iravani, “A neutral-point clamped converter system for direct-drive variable-speed wind power unit,” *Energy Conversion, IEEE Transactions on*, vol. 21, no. 2, pp. 596–607, 2006.

[29] K. Malekian, A. Shirvani, U. Schmidt, and W. Schufft, “Detailed modeling of wind power plants incorporating variable-speed synchronous generator,” in *Electrical Power Energy Conference (EPEC), 2009 IEEE*, Montreal, Canada, 2009.

[30] A. Eid, H. El-Kishky, M. Abdel-Salam, and M. El-mohandes, “On power quality of variable-speed constant-frequency aircraft electric power systems,” *Power Delivery, IEEE Transactions on*, vol. 25, no. 1, pp. 55–65, 2010.

[31] I. Moir and A. Seabridge, *Design and Development of Aircraft Systems*, ser. Aerospace Series. Wiley, 2012.

[32] V. Biagini, P. Zanchetta, M. Odavic, M. Sumner, and M. Degano, “Control and modulation of a multilevel active filtering solution for variable-speed constant-frequency more-electric aircraft grids,” *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 2, pp. 600–608, 2013.

[33] J. Ackermann, *Sampled-data Control Systems*. Berlin Heidelberg: Springer, 1985.

[34] R. E. Kalman, “A new approach to linear filtering and prediction problems,” *Transactions of the ASME - Journal of Basic Engineering*, vol. 82, pp. 35–45, 1960.

[35] M. Darouach, “Complements to full order observer design for linear systems with unknown inputs,” *Applied Mathematics Letters*, vol. 22, no. 7, pp. 1107–1111, 2009.

[36] S. Hui and S. Zak, “Low-order unknown input observers,” in *American Control Conference, Proceedings of the*, vol. 6, 2005, pp. 4192–4197.

[37] R. J. Patton, P. M. Frank, and R. N. Clarke, Eds., *Fault diagnosis in dynamic systems: theory and application*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1989.

[38] J. J. Gertler, *Fault Detection and Diagnosis in Engineering Systems*, 1st ed. Taylor & Francis, 1998.

[39] T. Wu, T. Camarano, J. Zumberge, M. Wolff, E. Lin, H. Huang, and X. Jia, “Electromagnetic design of aircraft synchronous generator with high power density,” in *Aerospace Sciences Meetings*. Nashville, USA: American Institute of Aeronautics and Astronautics, 2012.

[40] T. Lipo, *Analysis of Synchronous Machines*, 2nd ed. Taylor & Francis, 2012.

[41] M. Eremia and M. Shahidehpour, Eds., *Handbook of Electrical Power System Dynamics: Modeling, Stability, and Control*, ser. IEEE Press Series on Power Engineering. Wiley, 2013, vol. 92.

[42] J. Machowski, J. Bialek, and J. Bumby, *Power System Dynamics and Stability*. Wiley, 1997.

Appendix

A Proof of lemma 1

First, a ROO for the transformed system is constructed. After artificially expanding the system order to n, it is possible to combine $\hat{\boldsymbol{r}}(t)$ and $y(t)$. Finally, the system is transformed into original coordinates.

Given that system (1) is transformed to canonical coordinates using (7), the system equations are given by:

$$\dot{\boldsymbol{z}}(t) = \begin{bmatrix} 0 & 0 & \dots & -a_0 \\ 1 & 0 & \dots & -a_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 1 & -a_{n-1} \end{bmatrix} \boldsymbol{z}(t) + \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{bmatrix} u(t) \tag{a1a}$$

$$y(t) = \begin{bmatrix} 0 \dots 1 \end{bmatrix} \boldsymbol{z}(t) \ . \tag{a1b}$$

Here, $a_0, \dots, a_{n-1}$ are the coefficients of the characteristic polynomial. These equations can be partitioned according to the scheme in (5). With the resulting sub-matrices and a gain vector $\boldsymbol{f} \in \mathbb{R}^{n-1}$, the ROO estimating $\hat{\boldsymbol{z}}(t)$ reads:

$$\begin{aligned} \dot{\boldsymbol{\eta}}(t) &= (\boldsymbol{A}_{11} - \boldsymbol{f}\boldsymbol{A}_{21})\,\boldsymbol{\eta}(t) + (\boldsymbol{b}_1 - \boldsymbol{f}b_2)\,u(t) \\ &\quad + [(\boldsymbol{A}_{11} - \boldsymbol{f}\boldsymbol{A}_{21})\,\boldsymbol{f} + \boldsymbol{A}_{12} - \boldsymbol{f}A_{22}]\,y(t) \end{aligned} \tag{a2a}$$

$$\hat{\boldsymbol{r}}(t) = \boldsymbol{\eta}(t) + \boldsymbol{f}y(t) \tag{a2b}$$

$$\hat{\boldsymbol{z}}(t) = \begin{bmatrix} \hat{\boldsymbol{r}}(t) \\ y(t) \end{bmatrix} . \tag{a2c}$$

In order to facilitate the retransformation $\hat{\boldsymbol{x}}(t) = \boldsymbol{T}\hat{\boldsymbol{z}}(t)$, the observer state in canonical coordinates $\boldsymbol{\eta}(t)$ will be added by a zero component to increase its order to n:

$$\boldsymbol{\rho}^*(t) := \begin{bmatrix} \boldsymbol{\eta}(t) \\ 0 \end{bmatrix} . \tag{a3}$$

In the following, it is assumed that the value $\boldsymbol{\rho}^*(0)$ is chosen in the form of (a3). The ROO dynamic equation (a2a) is expanded to the order of n maintaining the last row of every matrix and vector to equal zero. Applying some matrix manipulations yields:

$$\dot{\boldsymbol{\rho}}^*(t) = \boldsymbol{N}_\mathrm{R}^*\boldsymbol{\rho}^*(t) + \boldsymbol{g}_\mathrm{R}^* u(t) + \boldsymbol{h}_\mathrm{R}^* y(t) \tag{a4a}$$
$$\hat{\boldsymbol{z}}(t) = \boldsymbol{\rho}^*(t) + \boldsymbol{l}_\mathrm{R}^* y(t) \ . \tag{a4b}$$

with

$$\boldsymbol{N}_\mathrm{R}^* = \boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T} - \tilde{\boldsymbol{\beta}}\boldsymbol{e}_n^\top - \begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}\boldsymbol{e}_{n-1}^\top \tag{a5a}$$
$$\boldsymbol{g}_\mathrm{R}^* = \left(\boldsymbol{I}_n - \begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}\boldsymbol{e}_n^\top\right)\boldsymbol{T}^{-1}\boldsymbol{b} \tag{a5b}$$
$$\boldsymbol{h}_\mathrm{R}^* = \begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}(a_{n-1} - f_{n-2}) + \begin{bmatrix}0\\\boldsymbol{f}\end{bmatrix} + \boldsymbol{a} \tag{a5c}$$
$$\boldsymbol{l}_\mathrm{R}^* = \begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix} \ . \tag{a5d}$$

Note that there is a degree of freedom given by the choice of $\tilde{\boldsymbol{\beta}}$ as the last column of $\boldsymbol{N}_\mathrm{R}^*$ is only related to the n-th element of $\boldsymbol{\rho}^*(t)$ which equals zero.

The resulting observer (a4) estimates the state vector in observable canonical form. To obtain the desired original state space vector the system has to undergo the transformation $\hat{\boldsymbol{x}}(t) = \boldsymbol{T}\hat{\boldsymbol{z}}(t)$. While $\boldsymbol{\rho}^*(t)$ denotes the vector in canonical form, $\boldsymbol{\rho}(t)$ represents the original states:

$$\dot{\boldsymbol{\rho}}(t) = \underbrace{\boldsymbol{T}\boldsymbol{N}_\mathrm{R}^*\boldsymbol{T}^{-1}}_{=:\boldsymbol{N}_\mathrm{R}}\boldsymbol{\rho}(t) + \underbrace{\boldsymbol{T}\boldsymbol{g}_\mathrm{R}^*}_{=:\boldsymbol{g}_\mathrm{R}}u(t) + \underbrace{\boldsymbol{T}\boldsymbol{h}_\mathrm{R}^*}_{=:\boldsymbol{h}_\mathrm{R}}y(t) \tag{a6a}$$
$$\hat{\boldsymbol{x}}(t) = \boldsymbol{\rho}(t) + \underbrace{\boldsymbol{T}\boldsymbol{l}_\mathrm{R}^*}_{=:\boldsymbol{l}_\mathrm{R}}y(t) \ . \tag{a6b}$$

In the following, the initial value condition (a3) and the actual observer (a4) are retransformed.

A.1 Mathematical relationships

First, helpful mathematical relationships are introduced. Premultiplying the definition of $\boldsymbol{s}_1$ in (4) with $\boldsymbol{Q}_\mathrm{B}$ gives:

$$\begin{bmatrix}\boldsymbol{c}\boldsymbol{s}_1 \ \boldsymbol{c}\boldsymbol{A}\boldsymbol{s}_1 \dots \boldsymbol{c}\boldsymbol{A}^{n-1}\boldsymbol{s}_1\end{bmatrix} = \begin{bmatrix}0\,0\dots1\end{bmatrix}^\top \ . \tag{a7}$$

Furthermore, the theorem of Cayley-Hamilton states that every $\boldsymbol{A}$ fulfils the characteristic equation:

$$0 = a_0\boldsymbol{I}_n + a_1\boldsymbol{A} + \dots + a_{n-1}\boldsymbol{A}^{n-1} + \boldsymbol{A}^n \ . \tag{a8}$$

Combining (a7) with (a8) yields:

$$\boldsymbol{c}\boldsymbol{A}^n\boldsymbol{s}_1 = -a_{n-1} \ . \tag{a9}$$

Moreover, (a8) directly shows that:

$$\boldsymbol{T}\boldsymbol{a} = \begin{bmatrix}\boldsymbol{s}_1 \ \boldsymbol{A}\boldsymbol{s}_1 \dots \boldsymbol{A}^{n-1}\boldsymbol{s}_1\end{bmatrix}\boldsymbol{a} = \boldsymbol{A}^n\boldsymbol{s}_1 \ . \tag{a10}$$

It is readily verified with (a7) and (a9) that the product $\boldsymbol{Q}_\mathrm{B}\boldsymbol{T}$ has the form:

$$\boldsymbol{Q}_\mathrm{B}\boldsymbol{T} := \boldsymbol{D} = \begin{cases} D_{ij} = 0 & i < n-j+1 \\ D_{ij} = 1 & i = n-j+1 \\ D_{ij} = -a_{n-1} & i = n-j+2 \end{cases} .$$

Then, relevant entries of the inverse of $\boldsymbol{D}$ are obtained as:

$$\boldsymbol{D}^{-1} = \begin{cases} D_{ij} = 0 & i > n-j+1 \\ D_{ij} = 1 & i = n-j+1 \\ D_{ij} = a_{n-1} & i = n-j \end{cases} . \tag{a11}$$

A.2 Transformation of $\boldsymbol{l}_\mathrm{R}^*$

First, (a6b) is considered. When multiplied with the transformation matrix $\boldsymbol{T}$, the second summand becomes:

$$\begin{aligned}\boldsymbol{l}_\mathrm{R} &= \begin{bmatrix}\boldsymbol{s}_1 \ \boldsymbol{A}\boldsymbol{s}_1 \dots \boldsymbol{A}^{n-1}\boldsymbol{s}_1\end{bmatrix}\begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix} \\ &= \left(f_0\boldsymbol{I}_n + \dots + f_{n-2}\boldsymbol{A}^{n-2} + \boldsymbol{A}^{n-1}\right)\boldsymbol{s}_1 \ .\end{aligned} \tag{a12}$$

A.3 Transformation of $\boldsymbol{N}_\mathrm{R}^*$

Transformation of (a5a) is performed employing (a9), (a11) and (a12):

$$\begin{aligned}\boldsymbol{N}_\mathrm{R} &= \boldsymbol{T}\left(\boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T} - \tilde{\boldsymbol{\beta}}\boldsymbol{e}_n^\top - \begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}\boldsymbol{e}_{n-1}^\top\right)\boldsymbol{T}^{-1} \\ &= \boldsymbol{A} - \boldsymbol{T}\tilde{\boldsymbol{\beta}}\boldsymbol{e}_n^\top\boldsymbol{T}^{-1} - \boldsymbol{T}\begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}\boldsymbol{e}_{n-1}^\top\underbrace{\boldsymbol{T}^{-1}\boldsymbol{Q}_\mathrm{B}^{-1}}_{=\boldsymbol{D}^{-1}}\boldsymbol{Q}_\mathrm{B} \\ &= \boldsymbol{A} - \boldsymbol{T}\tilde{\boldsymbol{\beta}}\boldsymbol{c} - \boldsymbol{T}\begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}(\boldsymbol{c}\boldsymbol{A} - a_{n-1}\boldsymbol{c}) \\ &= (\boldsymbol{I}_n - \boldsymbol{l}_\mathrm{R}\boldsymbol{c})\boldsymbol{A} - \underbrace{\boldsymbol{T}\left(\tilde{\boldsymbol{\beta}} - a_{n-1}\begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}\right)}_{=:\boldsymbol{\beta}}\boldsymbol{c} \ .\end{aligned} \tag{a13}$$

As $\tilde{\boldsymbol{\beta}}$ can be chosen arbitrarily, it is set to eliminate the second term and simplify (a13) to:

$$\boldsymbol{N}_\mathrm{R} = (\boldsymbol{I}_n - \boldsymbol{l}_\mathrm{R}\boldsymbol{c})\boldsymbol{A} \ . \tag{a14}$$

A.4 Transformation of $\boldsymbol{g}_{\mathrm{R}}^{*}$

Secondly, expression (a5b) is manipulated using (a12):

$$\begin{aligned}\boldsymbol{g}_{\mathrm{R}} &= \boldsymbol{T}\left(\boldsymbol{I}_n - \begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}\boldsymbol{e}_n^{\top}\right)\boldsymbol{T}^{-1}\boldsymbol{b}\\ &= \left(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{R}}\boldsymbol{c}\right)\boldsymbol{b}\ . \end{aligned} \tag{a15}$$

A.5 Transformation of $\boldsymbol{h}_{\mathrm{R}}^{*}$

The third summand in (a6a) is calculated from expression (a5c) by considering (a7), (a10) and (a12):

$$\begin{aligned}\boldsymbol{h}_{\mathrm{R}} =& \boldsymbol{T}\left(\begin{bmatrix}\boldsymbol{f}\\1\end{bmatrix}(a_{n-1} - f_{n-2}) + \begin{bmatrix}0\\\boldsymbol{f}\end{bmatrix} + \boldsymbol{a}\right)\\ =&\boldsymbol{l}_{\mathrm{R}}(a_{n-1} - f_{n-2}) + \boldsymbol{T}\begin{bmatrix}0\\\boldsymbol{f}\end{bmatrix} + \boldsymbol{A}^n\boldsymbol{s}_1\\ =& -\boldsymbol{l}_{\mathrm{R}}\left(f_0\underbrace{\boldsymbol{cAs}_1}_{=0} + \cdots + f_{n-2}\underbrace{\boldsymbol{cA}^{n-1}\boldsymbol{s}_1}_{=1} + \underbrace{\boldsymbol{cA}^{n}\boldsymbol{s}_1}_{=-a_{n-1}}\right)\\ &+ \underbrace{\left(f_0\boldsymbol{A} + f_1\boldsymbol{A}^2 + \cdots + f_{n-2}\boldsymbol{A}^{n-1} + \boldsymbol{A}^n\right)\boldsymbol{s}_1}_{=\boldsymbol{Al}_{\mathrm{R}}}\\ =& -\boldsymbol{l}_{\mathrm{R}}\boldsymbol{cA}\left(f_0 I + \cdots + f_{n-1}\boldsymbol{A}^{n-1}\right)\boldsymbol{s}_1 + \boldsymbol{Al}_{\mathrm{R}}\\ =& \left(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{R}}\boldsymbol{c}\right)\boldsymbol{Al}_{\mathrm{R}}\ . \end{aligned} \tag{a16}$$

A.6 Transformation of initial value condition

Equation (a3) sets a constraint on the choice of the initial value $\boldsymbol{\rho}^*(0)$. Because the ROO allows for an arbitrary initial value of the $(n-1)$-dimensional $\boldsymbol{\eta}(t)$, the effective restriction can be expressed as:

$$\boldsymbol{e}_n^{\top}\boldsymbol{\rho}^*(0) = \boldsymbol{e}_n^{\top}\begin{bmatrix}\boldsymbol{\eta}(0)\\0\end{bmatrix} = 0\ . \tag{a17}$$

This yields an equivalent constraint on $\boldsymbol{\rho}(0)$:

$$0 = \boldsymbol{e}_n^{\top}\boldsymbol{\rho}^*(0) = \boldsymbol{cT\rho}^*(0) = \boldsymbol{c\rho}(0)\ . \tag{a18}$$

Setting $\boldsymbol{\rho}(0) = \boldsymbol{0}$ or $\boldsymbol{\rho}(0) = \alpha\boldsymbol{l}_{\mathrm{R}}$ with $\alpha \in \mathbb{R}$ trivially fulfils the requirement.

The underlying reason is that the transformation $\boldsymbol{T}^{-1}\boldsymbol{\rho}(t) = \boldsymbol{\rho}^*(t)$ cannot be fulfilled with regular $\boldsymbol{T}^{-1}$ and an *arbitrary* $\boldsymbol{\rho}(t) \in \mathbb{R}^n$. Choosing $\boldsymbol{\rho}(0)$ in accordance with (a18) and observing that the last row of $\boldsymbol{N}_{\mathrm{R}}^*$ equals zero, it holds true for $\forall t$ that $\boldsymbol{\rho}(t)$ lies in the measurable subspace of $\mathbb{R}^n$ where this restriction does not apply.

B Proof of lemma 4

The observability matrix of the system in (32) has the following form and its regularity is required for the system to be fully observable:

$$\det\left(\begin{bmatrix}\boldsymbol{c} & 1\\ \boldsymbol{cA} & 0\\ \vdots & \vdots\\ \boldsymbol{cA}^n & 0\end{bmatrix}\right) = \det\left(\begin{bmatrix}\boldsymbol{cA}\\ \vdots\\ \boldsymbol{cA}^n\end{bmatrix}\right) = \det(\boldsymbol{Q}_{\mathrm{B}})\det(\boldsymbol{A})\ .$$

Since the original system is assumed to be observable, $\boldsymbol{Q}_{\mathrm{B}}$ is non-singular. Therefore, the regularity of $\boldsymbol{A}$ determines the observability of the extended system. However, if $\boldsymbol{A}$ posses an eigenvalue at zero and is thus singular, the first coefficient of its characteristic polynomial is $a_0 = 0$. In this case, the theorem of Caley Hamilton (a8) gives that the last row of $\boldsymbol{Q}_{\mathrm{B}}$ is linearly dependent on rows 2 until n. Therefore, the system is definitely not observable if $\boldsymbol{A}$ is singular.

C Proof of theorem 2

To obtain an observer of order n the explicit form of the ROO (9) is used. In the resulting dynamic equation the first n state estimates, namely $\boldsymbol{x}(t)$, can be separated from the $(n+1)$-th component $w(t)$.

In order to derive the observer, the $(n+1)$-dimensional observer gain is separated into two components $\boldsymbol{l}_{\mathrm{R}} = \begin{bmatrix}\boldsymbol{l}_{\mathrm{G}}\ l_2\end{bmatrix}^{\top}$ with $\boldsymbol{l}_{\mathrm{G}} \in \mathbb{R}^n,\ l_2 \in \mathbb{R}$. The ROO (9) applied to the system (32) then reads:

$$\begin{aligned}\begin{bmatrix}\dot{\boldsymbol{\rho}}_x(t)\\ \dot{\rho}_w(t)\end{bmatrix} =& \begin{bmatrix}(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\boldsymbol{A} & \boldsymbol{0}\\ -l_2\boldsymbol{cA} & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{\rho}_x(t)\\ \rho_w(t)\end{bmatrix}\\ &+ \begin{bmatrix}(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\boldsymbol{b}\\ -l_2\boldsymbol{cb}\end{bmatrix}u(t) + \begin{bmatrix}(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c})\boldsymbol{Al}_{\mathrm{G}}\\ -\boldsymbol{cAl}_{\mathrm{G}}l_2\end{bmatrix}y(t)\end{aligned} \tag{a1a}$$

$$\begin{bmatrix}\hat{\boldsymbol{x}}(t)\\ \hat{w}(t)\end{bmatrix} = \begin{bmatrix}\boldsymbol{\rho}_x(t)\\ \rho_w(t)\end{bmatrix} + \begin{bmatrix}\boldsymbol{l}_{\mathrm{G}}\\ l_2\end{bmatrix}y(t)\ . \tag{a1b}$$

Then, $\boldsymbol{\rho}_x(t)$ and $\hat{\boldsymbol{x}}(t)$ are separated from $w(t)$ to form:

$$\begin{aligned}\dot{\boldsymbol{\rho}}_x(t) =& \left(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c}\right)\boldsymbol{A\rho}_x(t)\\ &+ \left(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c}\right)\boldsymbol{b}u(t) + \left(\boldsymbol{I}_n - \boldsymbol{l}_{\mathrm{G}}\boldsymbol{c}\right)\boldsymbol{Al}_{\mathrm{G}}y(t)\end{aligned} \tag{a2a}$$

$$\hat{\boldsymbol{x}}(t) = \boldsymbol{\rho}_x(t) + \boldsymbol{l}_{\mathrm{G}}y(t)\ . \tag{a2b}$$

Unlike the ROO, there is no condition on the initial value $\boldsymbol{\rho}_x(0)$ because for an arbitrary $\boldsymbol{\rho}_x(0)$ there exists a $\rho_w(0)$

so that the state in (a1a) lies in the n-dimensional measurable subspace.

Jan Erik STELLET received the B.Sc. and M.Sc. degree in Electrical Engineering and Information Technology from the Karlsruhe Institute of Technology in 2010 and 2012 respectively. He is currently a Ph.D. student at the Karlsruhe Institute of Technology. Email: jan.stellet@student.kit.edu.

Tobias ROGG received the B.Sc. and M.Sc. degree in Electrical Engineering and Information Technology from the Karlsruhe Institute of Technology in 2011 and 2014 respectively. He is currently a Ph.D. student at the Swiss Federal Institute of Technology in Zurich. Email: roggt@hpe.ee.ethz.ch.