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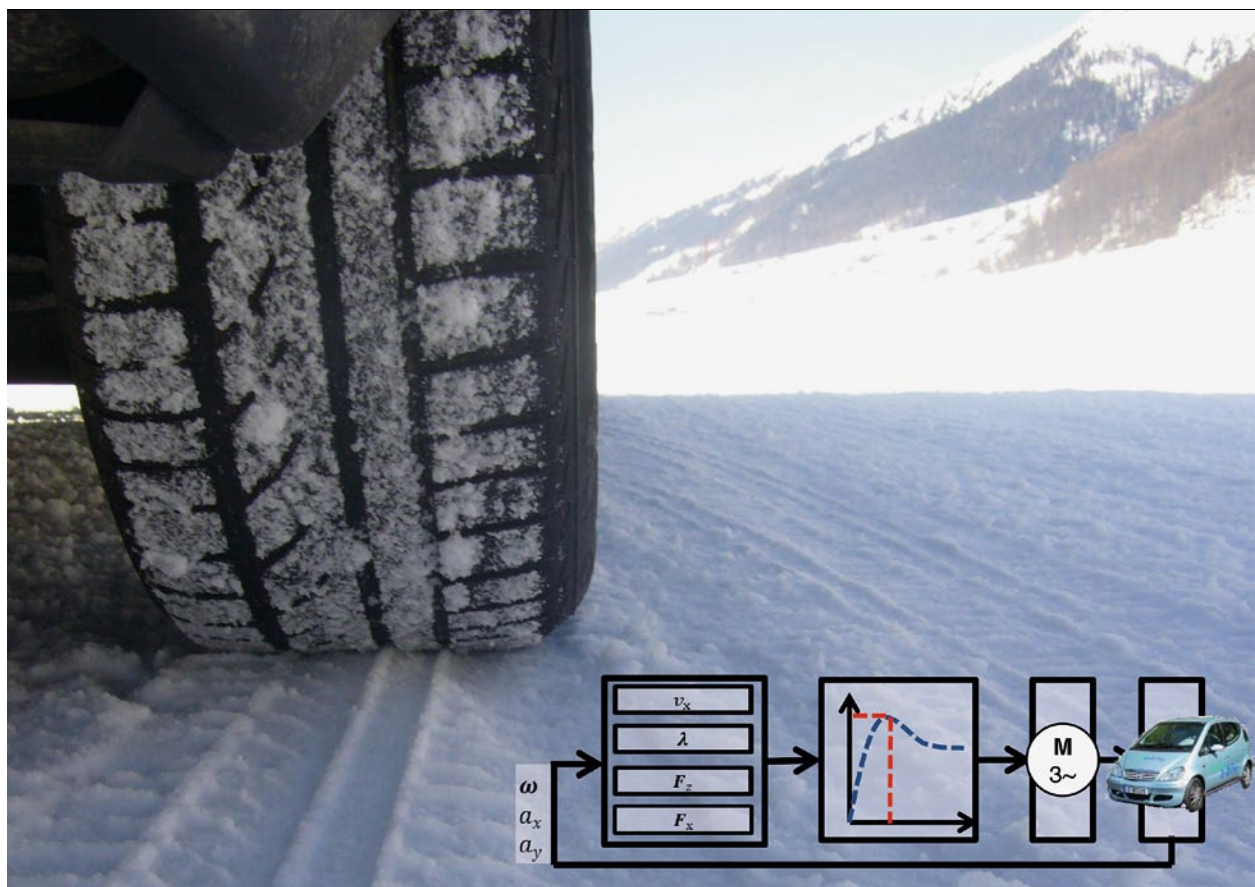


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MODEL-BASED TRACTION CONTROL FOR ELECTRIC VEHICLES

Optimal design and operating strategies for electric vehicles are investigated at the Karlsruhe Institute of Technology (KIT). This contribution details the design of a model-based traction control for an electrically powered vehicle.



1	MOTIVATION
2	CONTROL SYSTEM ARCHITECTURE
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4	ESTIMATION OF MU-SLIP-CURVE
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1 MOTIVATION

In order to increase vehicle safety, anti-slip control helps to avoid critical states in the tire-road contact. Critical situations occur if the maximum force which can be transmitted to the road is exceeded. This happens especially easily in wet or snowy road conditions. Consequences are blocked or slipping tires which severely affects the steerability of the vehicle. Therefore, it is important to monitor tire slip and control the driving torque accordingly.

Research on methods for traction control of electric vehicles is a current topic in science and industry. New concepts utilise the fast and precise controllability of the electric engine torque when compared to a combustion engine. This enables model-based approaches, where the relation between tire force and slip (mu-slip-curve) is continuously estimated [1, 2]. Controlling the tire slip to the optimum value is achieved using sliding mode control [3], adaptive control [4], model-predictive control [5] or fuzzy-control [6].

This contribution details the design of a model-based traction controller for an electric vehicle. Engine torque is generated by an asynchronous motor on the front axle. The tire slip setpoint value is derived from an estimate of the mu-slip-curve and chosen to maximise tire forces. Here, a novel linear parameterisation of the model is used which allows a more robust and precise estimation in comparison to previous approaches. Controlling the non-linear tire slip dynamics is performed with a sliding mode controller enhanced by conditional integration.

2 CONTROL SYSTEM ARCHITECTURE

The control system design based on works [7, 8] is shown in 1. There are separate modules for the estimation of the vehicle state variables, the estimation of the tire-road-friction characteristic and the actual tire slip controller.

Tire forces in longitudinal (F_x) and vertical (F_z) direction as well as driving speed v_x are derived from measurements of the wheel speeds ω and the vehicle accelerations a_x and a_y . The tire slip describes the wheel's movement relative to the vehicle speed. In the case of acceleration, it is defined as the normalised difference according to eq. (1) [9].

EQ. 1	$\lambda = \frac{\omega r - v_x}{\omega r}$
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In order to derive the reference value λ_0 , the normalised longitudinal tire force μ according to eq. (2) is calculated first.

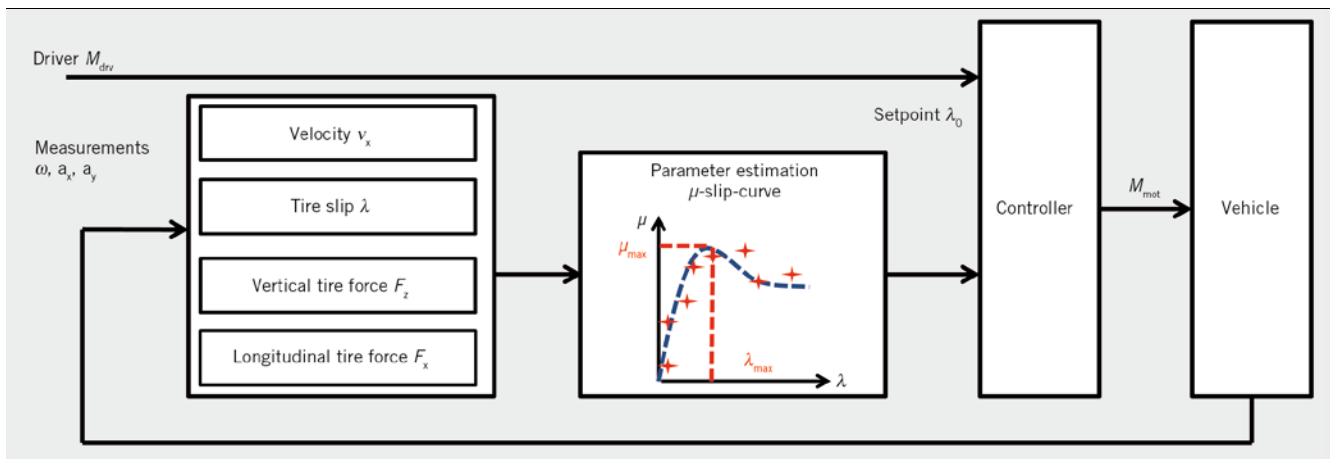
EQ. 2	$\mu = \frac{F_x}{F_z}$
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Conditions of tire (pressure, age) and road define a non-linear relation $\mu(\lambda)$ between tire-slip and transmittable force. Here, a parametric model is assumed for the quasistatic case and the model parameters are estimated. This gives the controller setpoint value λ_0 which allows for maximum normalised force transmission μ_{max} .

Under consideration of the engine torque M_{drv} that is requested by the driver, the tire slip is controlled to the setpoint value. The control output M_{mot} is then requested from the powertrain.

3 ESTIMATION OF VEHICLE STATES AND TIRE FORCES

On-line estimates of tire forces and tire slip are obtained from wheel speed and vehicle acceleration measurements [7, 10]. Assuming pure longitudinal dynamics, the system model according to eq. (3) is considered here. The dynamics of the tire forces depend on unknown external factors and are thus modelled with a Gaussian w as a random walk process.



1 Control system architecture

$$\text{EQ. 3} \quad \begin{cases} J_i \dot{\omega}_i = -r F_{x,i} + M_i i \in \{\text{FL}, \text{FR}\} \\ m \dot{v}_x = F_{x,VL} + F_{x,VR} - \frac{1}{2} c_w A \rho v_x^2 \\ \dot{F}_{x,i} = w_i i \in \{\text{FL}, \text{FR}\} \end{cases}$$

Estimation of vehicle speed v_x (for calculating the tire slip according to eq. (1)) and longitudinal tire forces $F_{x,i}$ on the front axle is performed using an extended Kalman filter. The measurement equation (4) is given by the speeds of the free-rolling wheels on the rear axle.

$$\text{EQ. 4} \quad v_x = \frac{1}{2} r (\omega_{RL} + \omega_{RR})$$

The vertical tire forces $F_{z,i}$ are determined by the vehicle's geometry with a simplified model of pitch and roll dynamics given in eq. (5) [7].

$$\text{EQ. 5} \quad \begin{cases} F_{z,i} = \frac{1}{2} \frac{l_{SH}}{l} mg - \left(\frac{1}{2} a_x \pm \frac{l_{SH}}{b} a_y \mp \frac{h_s}{b_g} a_x a_y \right) \\ \frac{h_s}{l} m \dot{i} \in \{\text{FL}, \text{FR}\} \end{cases}$$

4 ESTIMATION OF MU-SLIP-CURVE

4.1 QUASISTATIC MODEL OF TIRE FORCE TRANSMISSION

A crucial part of the system under control is given by the relation of force transmission and slip at the tires. In order to incorporate the varying conditions of road and tire, a parameterised model for the quasistatic case is employed.

One commonly used approach is the Burckhardt friction model [9] which describes the stationary normalised tire force as a function of tire slip according to eq. (6).

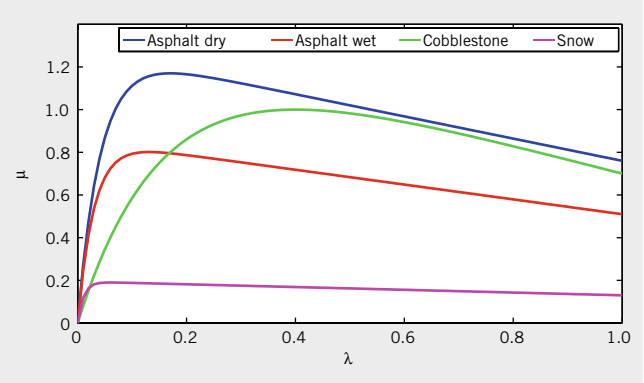
$$\text{EQ. 6} \quad \mu(\lambda) = c_1 (1 - \exp(-c_2 \lambda)) - c_3 \lambda$$

Four exemplary curves resulting from typical parameterisations are shown in 2. These are characterised by a maximum tire force transmission at $(\lambda_{max}, \mu_{max})$ for low slip values.

4.2 MODEL APPROXIMATION

The Burckhardt friction model from eq. (6) is non-linear with respect to the parameter c_2 . For parameter estimation methods or usage in adaptive controllers, it is however advantageous to have a linear relation between function values and model parameters as in eq. (7).

$$\text{EQ. 7} \quad \mu(\lambda) = \underbrace{[\psi_1(\lambda) \dots \psi_N(\lambda)]}_{\psi^T(\lambda)} \cdot \theta$$



2 Tire-force tire-slip curves according to the Burckhardt tire friction model with typical parameter values [7]

Approximating the Burckhardt friction model by a linear combination of basis functions according to eq. (8) is presented in [1, 2].

$$\text{EQ. 8} \quad \hat{\mu}(\lambda) = \sum_{i=1}^n \theta_i \exp(-w_i \lambda) + \theta_{n+1} - \theta_{n+2} \lambda$$

The parameters w_i that determine the n basis functions are derived by minimisation of the squared approximation error. Better approximation is obtained for higher n , however this also increases the dimension of the parameter vector θ which is eventually to be estimated on-line. Thus, a more accurate approximation is achieved at the cost of reduced estimator convergence speed. As a sensible compromise, [2] proposes to utilise $n = 3$ basis functions and therefore $N = 5$ model parameters in total.

Here, a novel set of basis functions according to eq. (9) is employed [12]. The optimal function parameters are determined by numerical solving of the non-linear optimisation problem. For the resulting approximation with $n = 3$ basis functions an improvement of 50% in the squared error is obtained compared to the previous solution from eq. (8). The optimal linear parameterisation is found as $w = [8,105 \ 27,547 \ 75,012]$.

$$\text{EQ. 9} \quad \hat{\mu}(\lambda) = \sum_{i=1}^n \theta_i [1 - \exp(-w_i \lambda)] - \theta_{n+1} \lambda$$

Apart from a reduction in the number of model parameters to $N = 4$ in this more accurate approximation, another advantage is that all friction curves include the origin as the Burckhardt friction model (6) does. This is not guaranteed in the approximation according to eq. (8). It is thus required to additionally incorporate this condition in the parameter estimation method [2]. Otherwise, convergence against implausible solutions may occur which is inherently prevented with the new scheme.

4.3 ON-LINE PARAMETER ESTIMATION

Given the time-discrete input values (λ_k, μ_k) that have been obtained as outlined in section 3, the model parameters of the linear parameterisation from eq. (9) have to be estimated using a recursive least-squares (RLS) estimator.

The parameter estimate $\hat{\theta}_k$ and the estimation error covariance P_k are updated in each time step k according to eq. (10) using a gain vector γ_k [13]. The impact of past measurements on the current estimate is continuously reduced by exponential weighting which is determined by a forgetting factor α ($0 < \alpha \leq 1$).

EQ. 10	$\begin{cases} \hat{\theta}_k = \hat{\theta}_{k-1} + \gamma_k \cdot [\mu_k - \Psi^T(\lambda_k) \hat{\theta}_{k-1}] \\ P_k = \frac{1}{\alpha} [P_{k-1} - \gamma_k \Psi^T(\lambda_k) P_{k-1}] \end{cases}$
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One commonly used approach is to calculate the gain γ_k on the basis of a user-defined constant forgetting factor. However, this causes problems in the application of tire road friction estimation. On the one hand, transition between different road surfaces is usually immediate and thus a fast adaptation of the estimator is desirable. Therefore, the estimator should have a short internal time horizon, that is a small value for α . On the other hand, the estimate should remain stable during periods of low excitation which is achieved with α close to 1.

In order to cope with these requirements, a variable forgetting factor α_k is employed here. Calculation of α_k and the corresponding gain vector γ_k is given by eq. (11) [14].

EQ. 11	$\begin{cases} \gamma_k = P_{k-1} \Psi(\lambda_k) \cdot [1 + \Psi^T(\lambda_k) P_{k-1} \Psi(\lambda_k)]^{-1} \\ \alpha_k = \max(\alpha_{\min}, 1 - \sum_0^{-1} [1 - \Psi^T(\lambda_k) \gamma_k] \\ [\mu_k - \Psi^T(\lambda_k) \theta_{k-1}]^2) \end{cases}$
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③ visualises results of an exemplary mu-slip-curve estimation. The vehicle accelerates from standstill on a road surface with friction coefficient $\mu_{\max} = 0,6$. After a few seconds, a different road type with $\mu_{\max} = 0,3$ is reached. In the case of a variable forgetting factor, adaptation to the new conditions is accomplished by a decrease in α . Most of the time however, the estimator's forgetting of infor-

mation is halted with $\alpha = 1$. In contrast, employing a constant $\alpha = 0,99$ causes an increase in estimator uncertainty and thus potential instability for lack of excitation.

5 CONTROLLER DESIGN

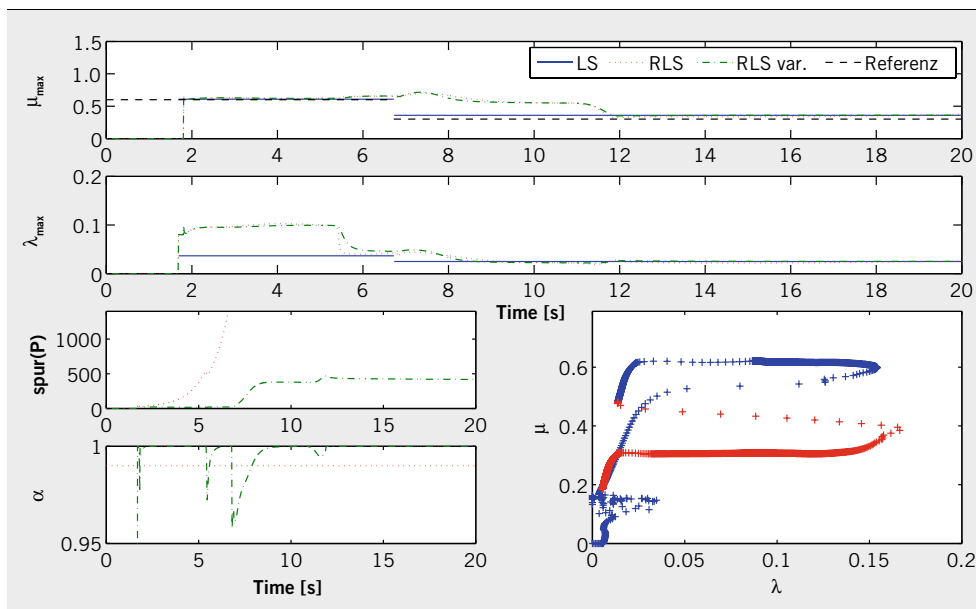
The previously described model identification of the mu-slip-curve is used to calculate the controller reference slip value λ_0 . Under the assumption of pure longitudinal dynamics, the point of maximum tire force transmission at λ_{\max} is chosen. Note that this is not necessarily an optimal choice in the case of both lateral and longitudinal dynamics. Here, the maximum longitudinal tire force is achieved for higher slip values. However, lateral force transmission decreases monotonously with tire slip which is undesired [9]. To cope with this dependency, one could choose the reference value λ_0 as a function of the side slip angle [15].

5.1 DYNAMIC MODEL

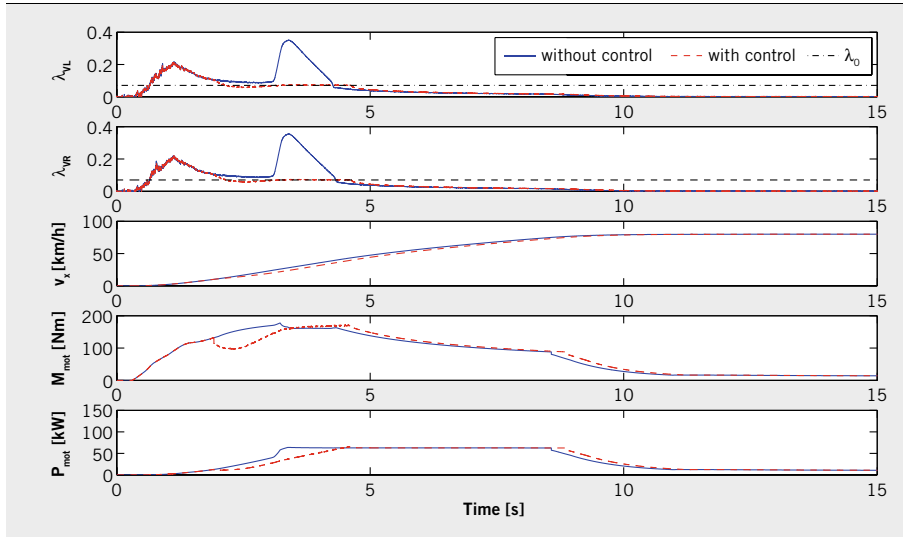
The controller calculates separate input torques $M_{\text{set},i}$, $i \in \{\text{FL}, \text{FR}\}$ for both tires. As only one central engine is assumed here, the motor torque M_{mot} is set in correspondence to the lower of both values. In situations with different road friction between both sides, the maximum force transmission is thus not reached. However, this select-low principle avoids the occurrence of undesired yaw moments which would have to be counteracted otherwise. Above all, the torque value M_{drv} that corresponds to the current throttle position limits the controller requested value.

The dynamic model of the system under control is derived from eq. (12) for both vehicle sides.

EQ. 12	$\begin{cases} J_r \dot{\omega} = -r F_x - r f_r F_z + M_{\text{set}} \\ m_{0,5} \dot{v}_x = F_x - \frac{1}{2} c_w A_{0,5} \rho v_x^2 \end{cases}$
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③ Recursive least-squares estimation with constant and variable forgetting factor; results from a non-recursive least-squares estimation are visualised for comparison; the estimator uncertainty is depicted in form of the sum of the diagonal elements (trace) of the covariance matrix



4 Tire slip, vehicle velocity, engine torque and engine power in acceleration manoeuvre

Here, the quantities per wheel are denoted as vehicle mass $m_{0,5}$ and cross-sectional area $A_{0,5}$. Under the assumption of v_x as a slowly time-varying parameter (compared to the tire dynamics), the differential equation (13) gives the dynamics of the tire slip λ [16].

EQ. 13	$\dot{\lambda} = -\gamma(\lambda) [\Psi(\lambda) - M_{set}]$ <p>with $\gamma(\lambda) = \frac{r(1-\lambda)^2}{J_r v_x}$</p> $\Psi(\lambda) = \left(\frac{J_r}{m_{0,5} r^2} \frac{1}{1-\lambda} + 1 \right) \mu(\lambda) F_z r$ $- \frac{J_r}{m_{0,5} r^2} \frac{1}{1-\lambda} \frac{1}{2} c_w A_{0,5} \rho v_x^2 + r f_r F_z$
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In practice, $\Psi(\lambda)$ is determined by time varying or uncertain parameters. For the approximation $\hat{\Psi}(\lambda)$ that is implemented in the control law, an error bound $\rho(\lambda)$ according to eq. (14) is calculated.

EQ. 14	$ \Psi(\lambda) - \hat{\Psi}(\lambda) \leq \rho(\lambda)$
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5.2 CONTROL LAW

A non-linear sliding mode controller (SMC) is employed here because it allows simple implementation and guaranteed stability under uncertain parameters.

The main principle is to define a stable trajectory (sliding surface) in state space that converges towards the reference value λ_0 . In order to achieve convergence from arbitrary initial values, a switching control law is designed [17]. An idealised control law is derived with the simple sliding surface $s(\lambda) = \lambda - \lambda_0$ and control gain K according to eq. (15).

EQ. 15	$M_{set} = \hat{\Psi}(\lambda) - \left(\frac{K}{\gamma(\lambda)} + \rho(\lambda) \right) \text{sgn}(s(\lambda))$ <p>with $\text{sgn}(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$</p>
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One major disadvantage of this approach is that perfect equality $\lambda - \lambda_0$ cannot be reached in practice. This causes a permanent switching of the discontinuous part of the control variable (chattering) [17] which is not allowed for mechanical actuators. In order to avoid this undesired effect, several modifications have been proposed.

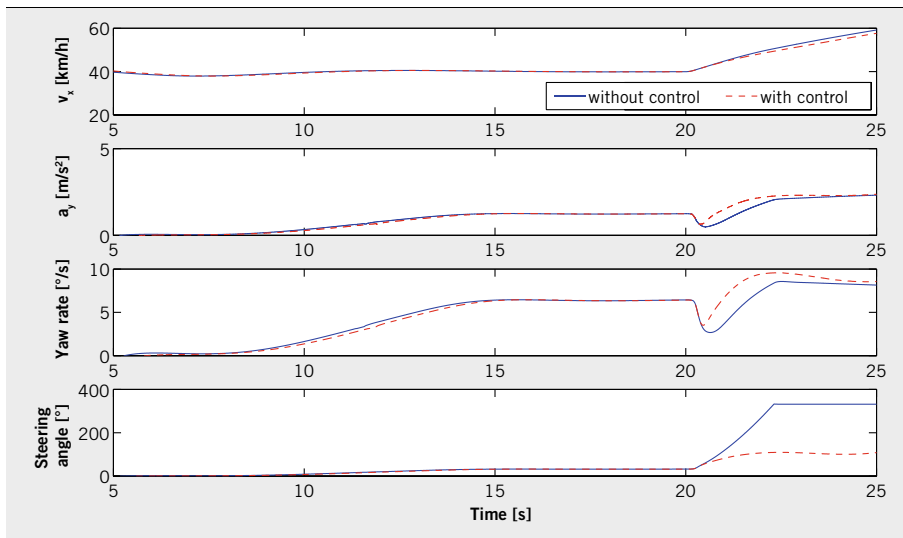
A linear function can be used to replace the idealised step function with a continuous approximation for a small tolerance band ϵ . However, this does not guarantee convergence of the controller within the tolerance band [17]. A more sophisticated approach that is employed here is to additionally enhance the sliding surface with a proportional-integral term. This extension termed Conditional Integration (SMC+CI) achieves convergence within the ϵ environment [3].

6 SIMULATION

The described control system is implemented into the simulation model of an electric vehicle in IPG carmaker. The vehicle features a central motor for propulsion of the front wheels. A comparison is made between the vehicle dynamics with and without control.

First, acceleration from standstill to $v_x = 80$ km/h under wet road conditions ($\mu_{max} = 0,7$) is considered. Results of this manoeuvre are shown in 4. The effectiveness of the controller can be seen from the reduced tire slip values. Integrating the electric engine power over the whole sequence gives that for an identical final velocity, a reduction in energy consumption of 3,3% is achieved.

A second simulation compares the vehicle dynamics when accelerating out of a steady state circular test. The vehicle enters a circle with constant radius $R = 100$ m, $\mu_{max} = 0,3$ at a velocity of $v_x = 40$ km/h. The time series shown in 5 indicate that steady state conditions with constant lateral acceleration, yaw rate and steering wheel angle are reached at $t = 15$ s. At $t = 20$ s the



5 Vehicle velocity, lateral acceleration, yaw rate and steering angle for steady-state circular test; the vehicle enters the circle at $t = 10$ s, steady-state conditions are reached at $t = 15$ s; switching to maximal throttle is performed at $t = 20$ s

maximum throttle is quickly applied. In the uncontrolled case, tire slip occurs and thus lateral tire forces drop. This results in a loss of steerability such that the vehicle does not follow the desired circular trajectory. It can be clearly seen that anti-slip control allows keeping the vehicle on track with only slightly increased steering effort.

7 CONCLUSION AND OUTLOOK

In this contribution, a model-based tire slip control based on an on-line estimation of the road friction coefficient has been developed. A novel linear parameterisation of the Burckhardt friction model was employed which yields improvements in accuracy, robustness and convergence speed of the parameter estimator. Controller design for the non-linear tire slip dynamic was studied using sliding-mode-control. Enhanced with a conditionally activated integral controller, the chattering effect is compensated for while exact convergence is maintained. Evaluation of the vehicle dynamics is performed in a realistic simulation environment and compared to the uncontrolled case. Reduced energy consumption during acceleration and improved lateral tire force transmission during cornering are found.

Future works could enhance the method by explicitly considering force transmission both in longitudinal and lateral direction. This is an especially relevant topic for vehicle concepts with individual electric engines for each wheel where the increased degrees of freedom could be used to optimise vehicle stability in critical situations.

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SYMBOLS	UNIT	DESCRIPTION
A	m^2	Cross-sectional area (frontal)
a_x	m/s^2	Longitudinal acceleration
a_y	m/s^2	Lateral acceleration
b	m	Vehicle width
c_w	–	Drag coefficient (frontal)
f	–	Rolling resistance coefficient
$F_{x,i}$	N	Longitudinal tire force at wheel i
$F_{z,i}$	N	Vertical tire force at wheel i
g	m/s^2	Gravitational acceleration
h_s	m	Height of vehicle's centre of gravity
i	–	Wheel front left: $i = FL$ Wheel front right: $i = FR$ Wheel rear left: $i = RL$ Wheel rear right: $i = RR$
J_i	$kg \cdot m^2$	Wheel moment of inertia
λ	–	Tire slip
l	m	Vehicle length
l_{SH}	m	Distance from vehicle's centre of gravity to rear axle
μ	–	Normalised longitudinal tire force
m	kg	Vehicle mass
M_{drv}	$N \cdot m$	Driver desired torque
M_{mot}	$N \cdot m$	Engine torque
$M_{set,i}$	$N \cdot m$	Controller set torque at wheel i
M_i	$N \cdot m$	Torque at wheel i
ρ	kg/m^3	Air density
r	m	Tire radius
θ	–	Vector of model parameters
v_x	m/s	Longitudinal velocity

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