Analysis and Performance Evaluation of Model Reference Adaptive Control

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1 First order system with one unknown parameter

1.1 Introduction

Adaptive Control techniques allow to control systems where certain system parameters are not known or change over time e. g. due to changing operating environment. This first example deals with a first order system with one unknown parameter, the system gain value b. The second parameter, the time constant a, is assumed to be known. In the second part of this article, the case when both parameters are unknown is considered.

One particular way of handling this problem is the technique of "Model Reference Adaptive Systems" (MRAS). This involves the definition of a *reference process* model whose dynamics in response to a *reference input* should be followed by the *plant process*. For the plant process with unknown parameters, a specific *control law* alters the the reference input signal in order for the plant's output signal to match the one of the reference model. This control law usually features time-variant controller parameter θ which reflect the algorithm's adaptation to the given plant system. The adaptation or learning component is incorporated by a time differential equation, the *update law*.

In all cases discussed here, the update law *directly* adjusts the controller parameters. This is called a *direct adaptive control approach*. The alternative would be to estimate the plant's system parameters first and *indirectly* determine a control law based on these estimations.

The reference model

$$Y_m(s) = G_m(s)\,R(s) = \frac{b_m}{s+a}R(s) \tag{1.1}$$

produces a model output y_m and from the process

$$Y_p(s) = G_p(s)\,U(s) = \frac{b}{s+a}U(s) \tag{1.2}$$

y_p results. Taking the difference of these two signals yields the tracking error e.

In order to ensure that the process dynamics

$$\dot{y}_p + a y_p = bu \tag{1.3}$$

match the desired reference dynamics

$$\dot{y}_m + a y_m = b_m r \tag{1.4}$$

the control law

$$u = r\theta \tag{1.5}$$

is an intuitive choice. Indeed, if the gain value θ is set to

$$\theta = \frac{b_m}{b} \tag{1.6}$$

then (1.3) becomes (1.4).

However, as the true value b is unknown, an adaptation mechanism for θ which is solely based on measurable quantities has to be found. Two different *update laws* will be derived and analysed in the following sections.

1.2 MIT rule

A commonly used update law is the "MIT rule". This adaptation mechanism is defined by

$$\frac{d}{dt}\theta = -\alpha e y_m \tag{1.7}$$

and incorporates the output error e as well as the reference model's output y_m.

To derive this update law, a cost function is defined first:

$$J = \frac{1}{2}e^2. \tag{1.8}$$

Essentially, the parameter θ is set to follow the "steepest descent" i. e. move along the negative gradient with a certain adaptation gain α:

$$\frac{d}{dt}\theta = -\alpha\frac{d}{d\theta}J \tag{1.9}$$

$$= -\alpha e\frac{d}{d\theta}e\left(\theta\right). \tag{1.10}$$

In order to calculate the *sensitivity derivative* $\frac{d}{d\theta}e\left(\theta\right)$, the system equations of model and plant process are inserted:

$$\frac{d}{dt}\theta = -\alpha e\frac{d}{d\theta}\left(y_p - y_m\right) \tag{1.11}$$

$$= -\alpha e\frac{d}{d\theta}\left(\mathcal{L}^{-1}\left\{G_p\left(s\right)U\left(s\right) - G_m\left(s\right)R\left(s\right)\right\}\right) \tag{1.12}$$

$$= -\alpha e\frac{d}{d\theta}\left(\mathcal{L}^{-1}\left\{G_p\left(s\right)\theta R\left(s\right) - G_m\left(s\right)R\left(s\right)\right\}\right) \tag{1.13}$$

$$= -\alpha e\frac{d}{d\theta}\left(\mathcal{L}^{-1}\left\{bG_m\left(s\right)\theta R\left(s\right) - G_m\left(s\right)R\left(s\right)\right\}\right) \tag{1.14}$$

$$= -\alpha eb\mathcal{L}^{-1}\left\{G_m\left(s\right)R\left(s\right)\right\} \tag{1.15}$$

$$= -\alpha eby_m\,. \tag{1.16}$$

Finally, merging α and b to a single gain value $\gamma = \alpha b$ yields the update law:

$$\frac{d}{dt}\theta = -\gamma e\frac{b_m}{s+a}r \tag{1.17}$$

$$= -\gamma e y_m\,. \tag{1.18}$$

1.3 Lyapunov rule

A second option for an update law is the "Lyapunov rule" which is given as

$$\frac{d}{dt}\theta = -\gamma e r \tag{1.19}$$

where r is the reference input signal for both systems.

In order to derive this formula, the following Lyapunov function is defined (similar to [1, p. 329]):

$$V = \frac{1}{2}\gamma e^2 + \frac{1}{2}b\left(\theta - \frac{b_m}{b}\right)^2\,. \tag{1.20}$$

The time derivative of V can be found as

$$\dot{V} = \gamma e\dot{e} + b\dot{\theta}\left(\theta - \frac{b_m}{b}\right) \tag{1.21}$$

and its negative definiteness would guarantee the tracking error going to zero along the system's trajectories.

Inserting the dynamic equations of model and plant process

$$\dot{y}_p = -y_p + \theta b r \tag{1.22}$$

$$\dot{y}_m = -y_m + b_m r \tag{1.23}$$

yields

$$\begin{aligned} \dot{V} &= \gamma e \left(\dot{y}_p - \dot{y}_m\right) + b\dot{\theta}\left(\theta - \frac{b_m}{b}\right) && (1.24)\\ &= \gamma e \left(-y_p + \theta b r + y_m - b_m r\right) + \dot{\theta}\left(b\theta - b_m\right) && (1.25)\\ &= \gamma e \left(-\left(y_p - y_m\right) + r\left(b\theta - b_m\right)\right) + \dot{\theta}\left(b\theta - b_m\right) && (1.26)\\ &= -\gamma e^2 + \left(\gamma e r + \dot{\theta}\right)\left(b\theta - b_m\right)\ . && (1.27) \end{aligned}$$

The last line gives the following condition for the Lyapunov function's derivative to be negative definite and thus the update law:

$$\frac{d}{dt}\theta = -\gamma e r\ . \tag{1.28}$$

Note that this result is similar to the MIT rule in equation (1.17) with the only difference being an additional filter operation of r with the reference model dynamics.

1.4 Summary

Deriving the MIT rule and Lyapunov adaptation law for a first order system with unknown gain parameter b reveals certain similarities as well as differences.

In both cases, the update law for the parameter θ is defined by its derivative $\frac{d}{dt}\theta$. This derivative is the product of a adaptation gain factor $-\gamma$ and two of the system's signals.

Regarding the MIT rule, these two factors are the error signal e and the reference model's output y_m. This relationship is derived using a cost function $J(e)$ (equation (1.8)) and varying the parameter in the direction of the negative gradient.

In contrast, the Lyapunov rule is established using the approach of a Lyapunov function (equation (1.20)) whose negative definiteness should be maintained by a suitable update law. This results in the update law being the product of the adaptation gain $-\gamma$, the error signal e and the reference input signal r. Therefore, the signal y_m as found in the MIT rule, is exchanged with r in the otherwise identical Lyapunov adaptation mechanism. However, y_m can be expressed as

$$y_m = \frac{b_m}{s+a} r \tag{1.29}$$

and thus the essential difference is an additional filter term in the MIT rule.

2 First order system with two unknown parameters

2.1 Introduction

In contrast to the previous section, the *true plant process* now contains two unknown parameters, a and b:

$$Y_p(s) = G_p(s)\,U(s) = \frac{b}{s+a}U(s)\ . \tag{2.1}$$

The following transfer function describes the *reference model*:

$$Y_m(s=)\,G_m(s)\,R(s) = \frac{b_m}{s+a_m}R(s)\ . \tag{2.2}$$

Again, the process dynamics

$$\dot{y}_p + a y_p = bu \tag{2.3}$$

should follow the reference dynamics

$$\dot{y}_m + a_m y_m = b_m r\ . \tag{2.4}$$

Intuitively, the control law should also include two parameters θ_1, θ_2.

Chosing the *control law*

$$u = r\theta_1 - y_p\theta_2\ . \tag{2.5}$$

and inserting it into equation (2.3) yields

$$\dot{y}_p + a y_p = b\left(r\theta_1 - y_p\theta_2\right) \tag{2.6}$$

$$\dot{y}_p + (a + b\theta_2)\,y_p = b\theta_1 r\ . \tag{2.7}$$

If θ_1, θ_2 were to be found as

$$\theta_1 = \frac{b_m}{b} \tag{2.8}$$

and

$$\theta_2 = \frac{a_m - a}{b} \tag{2.9}$$

then equation (2.7) would be identical to the reference dynamics. The system structure using this control law is illustrated in the block diagram in figure 2.1.

The first part of control law (2.5) is the same as obtained in the previous part. Setting $\theta_2 = 0$ therefore reduces the adaptation mechanism to the already known concept

$$u = r\theta_1\ . \tag{2.10}$$

However, the presence of a second unknown parameter a prevents the adaptation mechanism to work successfully without considering θ_2. Due to the second parameter (here: $a = 1.0$) being unknown and the reference model time constnat (here: $a_m = 2.0$) not equalling the true value, different system behaviour results as depicted in figure (2.2). In the first case the error signal converges to zero, showing damped oscillations. The reduced control law however causes undamped oscillations as the reduced control law is not capable of adapting the process to the reference model.

The properties of this model with two unknown parameters are further detailed in the following section, where the update laws are derived.

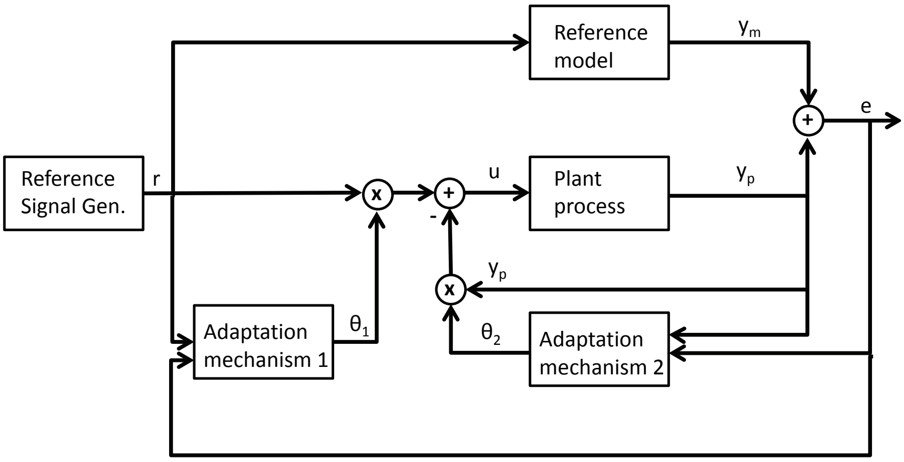

Figure 2.1: Block diagram of adaptive control system.

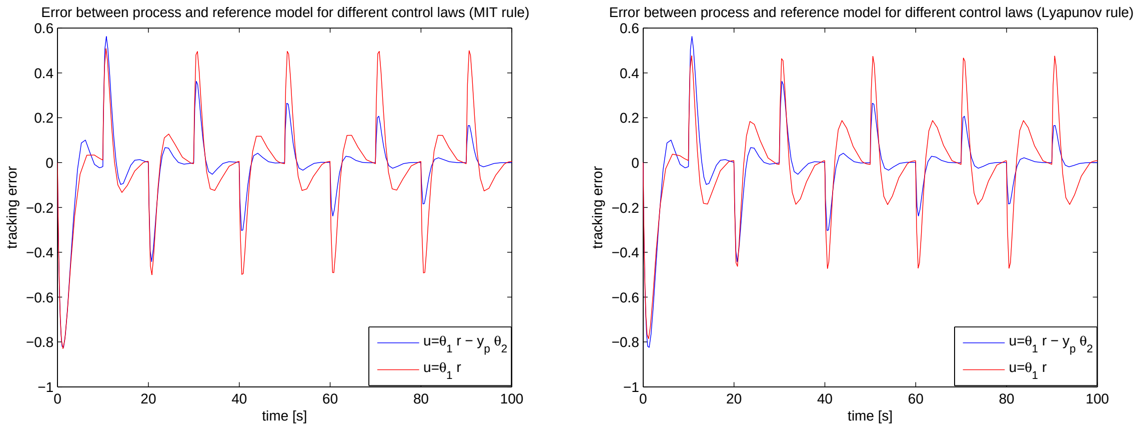

Figure 2.2: The error between model and plant is driven to zero by the control law $u = r\theta_1 - y_p\theta_2$ but remains periodic if $u = r\theta_1$. (Left: MIT rule, right: Lyapunov rule).

2.2 MIT rule

A parameter update law for the system is now derived based on the MIT rule for both θ_1 and θ_2.

As in the previous section (equation (1.10)) , the approach

$$\frac{d}{dt}\theta = -\alpha e \frac{d}{d\theta} e(\theta) \tag{2.11}$$

is used in order to find an update law for θ_1 and θ_2.

First of all, the plant process is given as

$$\dot{y}_p + a y_p = bu \; . \tag{2.12}$$

Inserting the control law defined in equation (2.5) and using $s(\cdot) := \frac{d}{dt}(\cdot)$ as differential operator yields:

$$\dot{y}_p + a y_p = b\left(r\theta_1 - y_p\theta_2\right) \tag{2.13}$$

$$y_p\left(\frac{d}{dt} + a + b\theta_2\right) = b\theta_1 r \tag{2.14}$$

$$y_p = \frac{b\theta_1}{s + a + b\theta_2} r \; . \tag{2.15}$$

Calculating the sensitivity derivatives $\frac{d}{d\theta} e(\theta)$ is performed using equation (2.15):

$$\frac{d}{d\theta_1} e = \frac{d}{d\theta_1}\left(y_p - y_m\right) \tag{2.16}$$

$$= \frac{d}{d\theta_1} y_p \tag{2.17}$$

$$= \frac{d}{d\theta_1}\left(\frac{b\theta_1}{s + a + b\theta_2} r\right) \tag{2.18}$$

$$= \frac{b}{s + a + b\theta_2} r \; . \tag{2.19}$$

And similarly:

$$\frac{d}{d\theta_2} e = \frac{d}{d\theta_2}\left(y_p - y_m\right) \tag{2.20}$$

$$= \frac{d}{d\theta_2}\left(\frac{b\theta_1}{s + a + b\theta_2} r\right) \tag{2.21}$$

$$= -\frac{b^2\theta_1}{\left(s + a + b\theta_2\right)^2} r \tag{2.22}$$

$$= -\frac{b}{s + a + b\theta_2} \cdot \frac{b\theta_1}{s + a + b\theta_2} r \tag{2.23}$$

$$= -\frac{b}{s + a + b\theta_2} \cdot y_p \; . \tag{2.24}$$

However, equation (2.19) and (2.24) still contain the unknown true parameters a and b. Therefore, an approximation has to be made for these terms. *Perfect model following* is achieved by comparison of the plant model from equation (2.14) with the reference model [1, p. 327]:

$$y_p\left(\frac{d}{dt} + a + b\theta_2\right) = b\theta_1 r \tag{2.25}$$

$$y_m\left(\frac{d}{dt} + a_m\right) = b_m r \; . \tag{2.26}$$

Chosing

$$\theta_1 b = b_m \qquad\qquad a + b\theta_2 = a_m \tag{2.27}$$

results in these equations to match and the tracking error to vanish.

Inserting these approximations into the sensitivity derivatives yields

$$\frac{d}{d\theta_1} e = \frac{b}{s + a_m} r \tag{2.28}$$

$$= \frac{b}{a_m} \frac{a_m}{s + a_m} r \tag{2.29}$$

and

$$\frac{d}{d\theta_2} e = -\frac{b}{s + a_m} y_p \tag{2.30}$$

$$= -\frac{b}{a_m} \frac{a_m}{s + a_m} y_p \,. \tag{2.31}$$

Finally, inserting these expressions into the the approach (2.11) and merging factors into a single adaptation gain $\gamma = \alpha \frac{b}{a_m}$ as in [2, p. 623] gives the update laws for θ_1 and θ_2:

$$\frac{d}{dt}\theta_1 = -\alpha e \frac{b}{a_m} \frac{a_m}{s + a_m} r \tag{2.32}$$

$$= -\gamma e \frac{a_m}{s + a_m} r \tag{2.33}$$

$$\frac{d}{dt}\theta_2 = \alpha e \frac{b}{a_m} \frac{a_m}{s + a_m} y_p \tag{2.34}$$

$$= \gamma e \frac{a_m}{s + a_m} y_p \,. \tag{2.35}$$

Now, that the control law and update law have been analysed, the effect of changes in parameter values is investigated in the next section.

2.3 Lyapunov rule

In order to derive an update law using Lyapunov theory for the case of two unknown variables, the following Lyapunov function is defined [3]:

$$V = \frac{1}{2}\gamma e^2 + \frac{1}{2b}\left(b\theta_1 - b_m\right)^2 + \frac{1}{2b}\left(b\theta_2 + a - a_m\right)^2 + \,. \tag{2.36}$$

The time derivative of V can be found as

$$\dot{V} = \gamma e \dot{e} + \dot{\theta}_1 \left(b\theta_1 - b_m\right) + \dot{\theta}_2 \left(b\theta_2 + a - a_m\right) \tag{2.37}$$

and its negative definiteness would guarantee the tracking error going to zero along the system's trajectories.

Inserting the dynamic equations of model and plant process

$$\dot{y}_p = -a y_p + b\left(r\theta_1 - y_p\theta_2\right) \tag{2.38}$$

$$\dot{y}_m = -a_m y_m + b_m r \tag{2.39}$$

yields

$$\dot{V} = \gamma e\left(\dot{y}_p - \dot{y}_m\right) + \dot{\theta}_1\left(b\theta_1 - b_m\right) + \dot{\theta}_2\left(b\theta_2 + a - a_m\right) \tag{2.40}$$

$$= \gamma e\left(-a y_p + b\left(r\theta_1 - y_p\theta_2\right) + a_m y_m - b_m r - a_m y_p + a_m y_p\right) + \dot{\theta}_1\left(b\theta_1 - b_m\right) + \dot{\theta}_2\left(b\theta_2 + a - a_m\right) \tag{2.41}$$

$$= \gamma e\left(-a_m e - y_p\left(b\theta_2 + a - a_m\right) + r\left(b\theta_1 - b_m\right)\right) + \dot{\theta}_1\left(b\theta_1 - b_m\right) + \dot{\theta}_2\left(b\theta_2 + a - a_m\right) \tag{2.42}$$

$$= -\gamma a_m e^2 + \left(\gamma e r + \dot{\theta}_1\right)\left(b\theta_1 - b_m\right) + \left(-\gamma e y_p + \dot{\theta}_2\right)\left(b\theta_2 + a - a_m\right) \,. \tag{2.43}$$

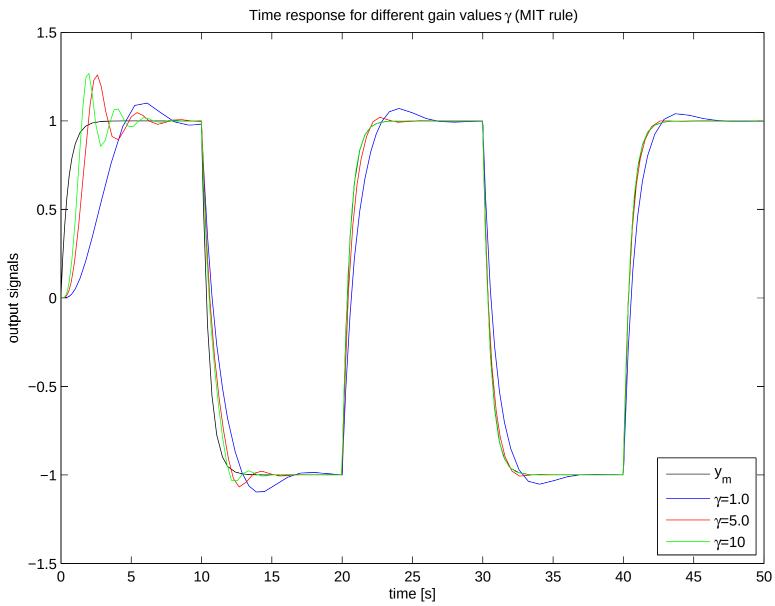

Figure 2.3: MIT rule: Time response for varying adaptation gain values γ.

The last line gives the following condition for the Lyapunov function's derivative to be negative definite and thus the update law:

$$\frac{d}{dt}\theta_1 = -\gamma e r \; . \tag{2.44}$$

$$\frac{d}{dt}\theta_2 = \gamma e y_p \; . \tag{2.45}$$

2.4 Experimental performance evaluation

This section details an experimental performance evaluation of the adaptive controller. It is investigated how different adaptation gain values affect the system behaviour, both for the MIT rule and the Lyapunov adaptation strategy [2, 4, 5]. Secondly, changes in the desired system response as defined by the reference model's parameters a_m, b_m are investigated.

Analysing the adaptive controllers was performed using a square wave signal with a time period of 20s and unit amplitude. This can be interpreted as a repeted step function with each step lasting for a duration of 10 s. While it is common to use a single step function to examine standard controller behaviour (step response), the nature of *adaptive* controllers makes it convenient to employ square wave signals and observe how the controller adapts over time.

In all cases, the true plant parameters were set to $a = 1.0$ and $b = 0.5$.

2.4.1 MIT rule

At first, the adaptation gain γ is varied and the resulting influence on the system's time response is analysed. The results of an experiment with gain values 1.0, 5.0 and 10 are shown in figure 2.3. Moreover, table 2.1 gives the approximate overshoot and 5% settling time as observed in these simulations.

The following observations can be made from these results:

- Increasing the gain value reduces the settling time. Thus it takes less time for the plant's output signal to approach the desired reference model's response.

Table 2.1: MIT rule: Settling time and overshoot of first step input for varying adaptation gains (approximations from figure 2.3).

	overshoot (1. step)	peak time (1. step)	5% settling time (1. step)
y_m (reference)	-	-	1.48 s
$\gamma = 1.0$	10.7%	5.79 s	7.04 s
$\gamma = 5.0$	26.2%	2.55 s	4.52 s
$\gamma = 10$	27.5%	1.89 s	4.22 s

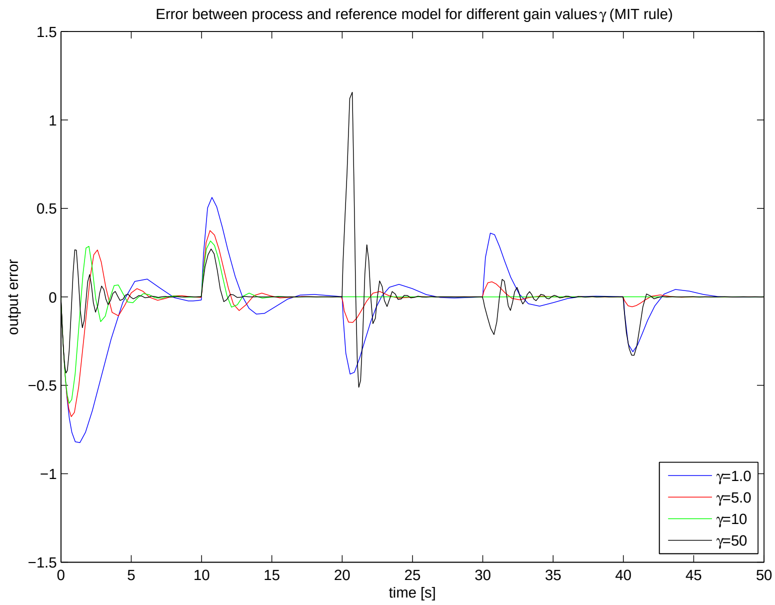

Figure 2.4: MIT rule: Tracking error for varying adaptation gain values γ.

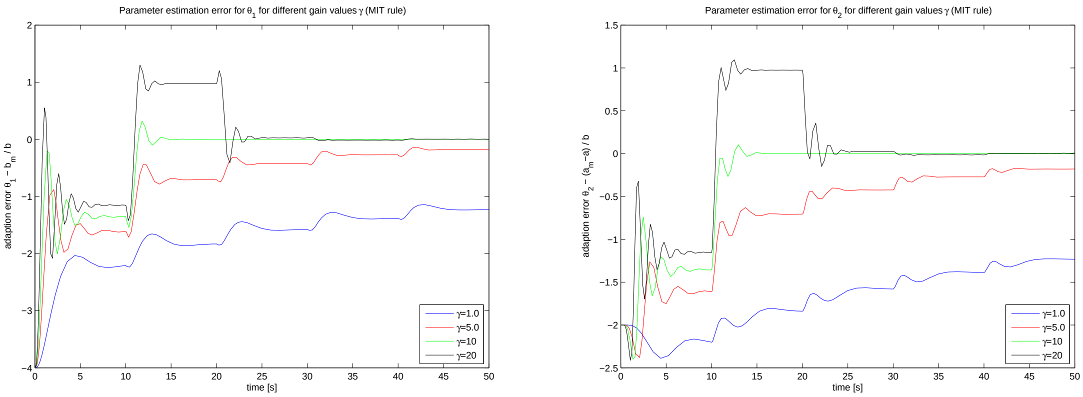

Figure 2.5: MIT rule: Parameter estimation error for θ_1 (left) and θ_2 (right) for varying adaptation gain values γ.

Table 2.2: Lyapunov rule: Settling time and overshoot of first step input for varying adaptation gains (approximations from figure 2.6).

	overshoot (1. step)	peak time (1. step)	5% settling time (1. step)
y_m (reference)	-	-	1.48 s
$\gamma = 1.0$	13.5%	5.32 s	6.67 s
$\gamma = 5.0$	28.8%	2.21 s	5.30 s
$\gamma = 10$	27.0%	1.61 s	3.90 s

- At the same time, oscillations and overshoot occur which are not present in the reference model's output (first order system). Increasing the gain value increases the frequency of these oscillations and also results in higher overshoot.

The tracking error between the reference system and the process over time is displayed in figure 2.4. As the time responses already indicated, higher gain values result in shorter settling time for the error signal but at the expense of higher overshoot and may cause undesired behaviour (parameter estimation not converging).

It is then analysed, how the parameter estimates θ_1 and θ_2 relate to the ideal values $\theta_1 = \frac{b_m}{b}$ and $\theta_2 = \frac{a_m - a}{b}$. It can be seen from figure 2.5 that the estimation error decreases over time while the controller "adapts" to the system. Although higher gain values may reduce the time needed for the parameter estimate to converge to the true value, this may also cause overshoot or even no convergence.[1]

In conclusion, it thus depends on the specific application's requirements whether a smaller overshoot or shorter settling times are favoured. In any case, the possibility of unwanted instability when increasing γ needs to be considered.

2.4.2 Lyapunov rule

Again, adaptation gain γ is varied firstly and the resulting influence on the system's time response is examined. Conducting the same experiment as in previous section with gain values 1.0, 5.0 and 10 gives the results which are visualised in figure 2.6. Moreover, table 2.2 states the approximate overshoot and 5% settling time.

The tracking error between the reference system and the process over time is displayed in figure 2.7. As the time responses already indicated, higher gain values result in shorter settling time for the error

[1] Setting $\gamma = 50$ results in no convergence at all, in order to ensure readability of the plots, these graphs are is not depicted here.

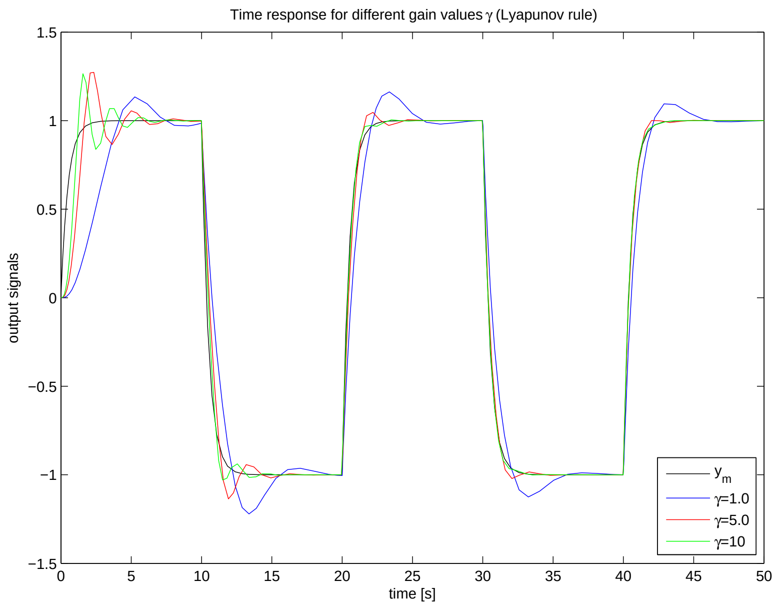

Figure 2.6: Lyapunov rule: Time response for varying adaptation gain values γ.

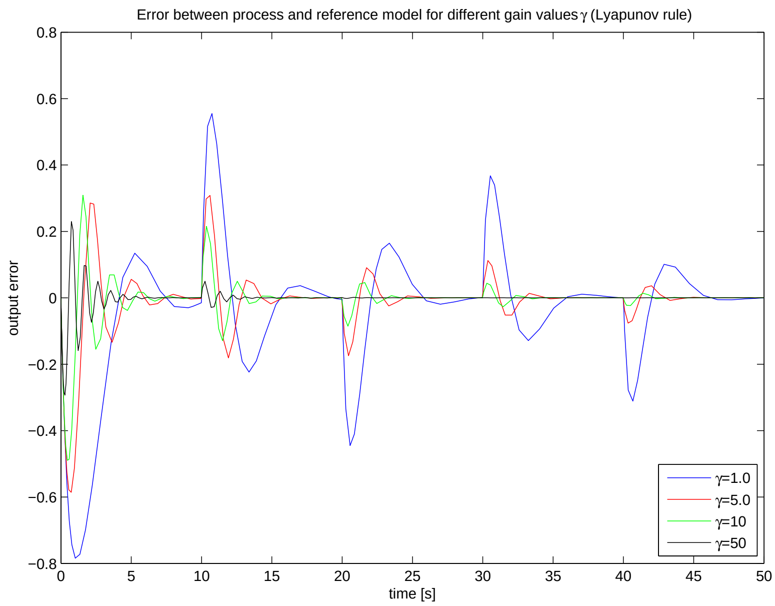

Figure 2.7: Lyapunov rule: Tracking error for varying adaptation gain values γ.

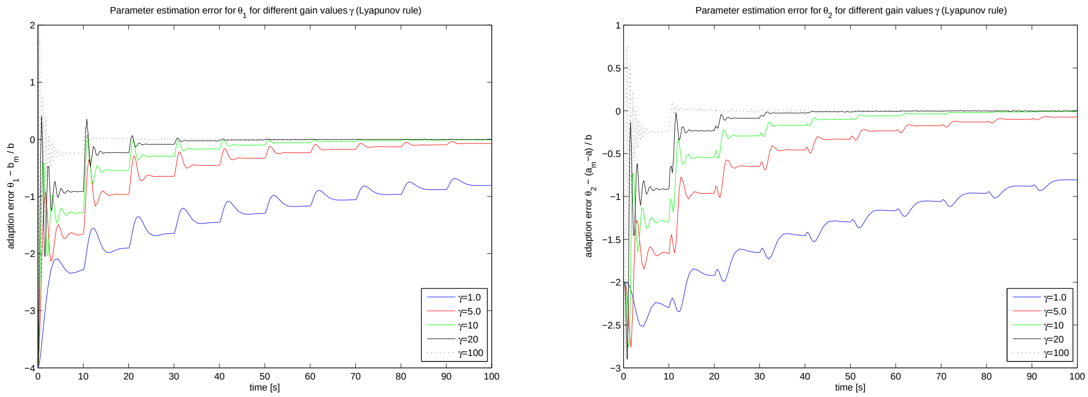

Figure 2.8: Lyapunov rule: Parameter estimation error for θ_1 (left) and θ_2 (right) for varying adaptation gain values γ.

signal. In contrast to the result for the MIT rule as seen in figure 2.4, no excessive overshoot can be observed when setting $\gamma = 50$.

Secondly the parameter estimation error for θ_1 and θ_2 is recorded and shown in figure 2.8. As with the results obtained using the MIT rule, the controller eventually adapts to the true values and the estimation error converges to zero. The most significant difference in these results is that no instability can be observed. Even when setting $\gamma = 100$, convergence is maintained though with increased overshoot. Using the MIT rule on the other hand yields unstable behaviour for these high gain values.

2.4.3 Reference model parameters

Secondly, the influence of changing the reference model's parameters a_m and b_m is examined.

The first parameter a_m is the time constant of the process and higher values result in the reference system's step response approaching the end value faster. Changing the reference model is equivalent to requesting a different time response for the plant process, which is set to $a = 1.0$. The objective of this experiment is therefore to investigate whether the adaptive controller will successfully speed up the plant's response as well.

Figure 2.9 shows the time responses of plant process y_p and reference model y_m for the default value $a_m = 2.0$. Only a slight difference between MIT rule and Lyapunov rule can be noticed with the MIT rule showing less overshoot.

Increasing the time constant to $a_m = 10$ results in the expected shorter rise time for y_m as seen in figure 2.10. For both update laws the plot of y_p shows a reduced rise time as well, but significant overshoot prevents it from reaching the desired behaviour of shorter settling time. Therefore, the adaptation mechanism itself is not fast enough for adapting to very small time constant. Almost no difference can be seen between MIT and Lyapunov adaptation strategies.

On the other hand, reducing the parameter value to $a_m = 0.5$ and $a_m = 0.3$ and thus increasing the time constant results in the behaviour which is depicted in figures 2.11 and 2.12. In these two cases, the MIT rule (red) does not yield successful adaptation whereas using the Lyapunov rule (green) shows excellent results and almost perfect following of the reference trajectory (blue).

Further experiments with different value combinations reveal that significant differences between MIT and Lyapunov rule occur if a_m is considerably lower than a. On the other hand, in the case of $a_m > a$ the results are almost similar. Taking into account the similarities between the two different update laws in equations (2.33) and (2.44) as well as (2.35) and (2.45) gives an idea of why this might be the case: The only difference is that the MIT rule features an additional dynamic term which resembles a low-pass structure. As this filter term incorporates a_m as its time constant, lower values than a therefore result in a delay which is greater than the process' delay. This suggests that in the example discussed

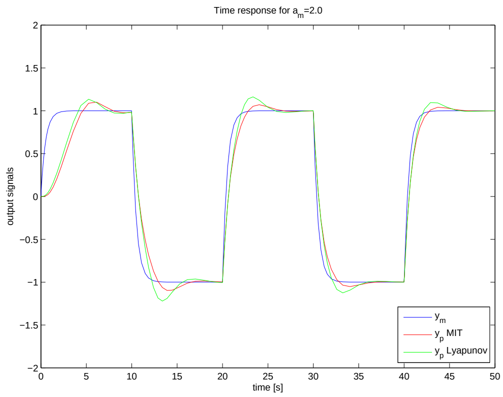

Figure 2.9: Time responses for $a_m = 2.0$.

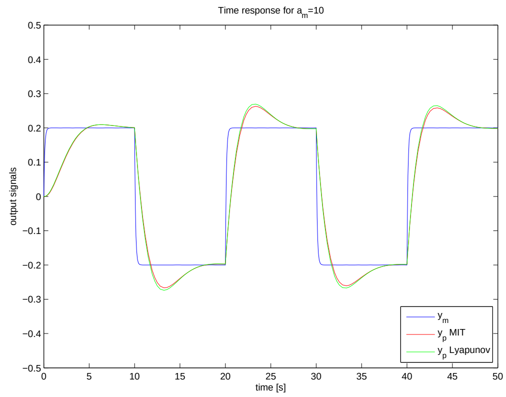

Figure 2.10: Time responses for $a_m = 10$.

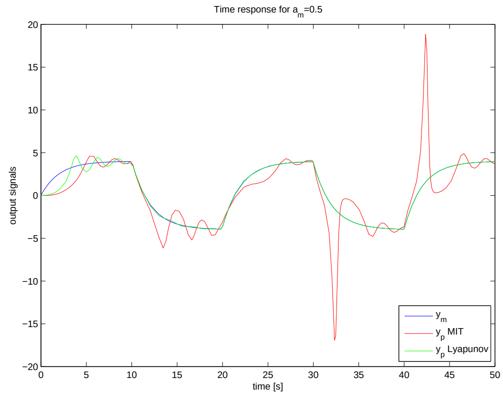

Figure 2.11: Time response for $a_m = 0.5$.

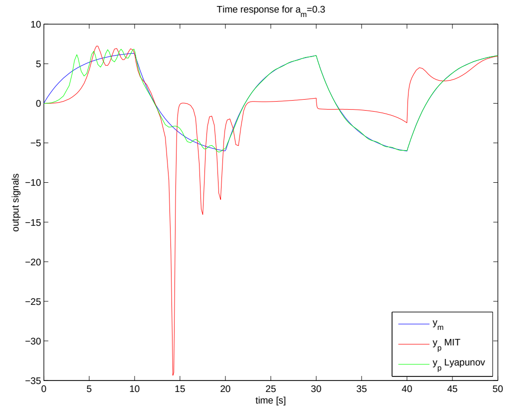

Figure 2.12: Time response for $a_m = 0.3$.

here the MIT rule update law is conceptually slower than required if $a_m < a$.

The second parameter b_m determines the system gain and the resulting output error is examined for different values of b_m (the plant process features $b = 0.5$). It can be seen from the results in figures 2.13 to 2.16 that higher values for this *reference model gain* parameter cause similar behaviour as already seen when varying the *adaptation gain*.

It is noteworthy to point out that although the MIT rule may lead to instability for high gain values as seen in figure 2.16 ($b_m = 50$), it also shows advantages in the case of $b_m = 10$ (figure 2.15). In contrast to the Lyapunov rule, the oscillations in the time response are of smaller amplitude. This can again be related to the additional filter term in the MIT rule which damps oscillations of certain frequencies in the output signal.

2.5 Summary

It can be concluded that although an adaptive controller may control a plant's behaviour to match the one of a predefined reference model in theory, several limitations underly the practical realisation. Rigorous testing and simulation with relevant input signals and parameter choices are necessary in order to ensure satisfactory behaviour.

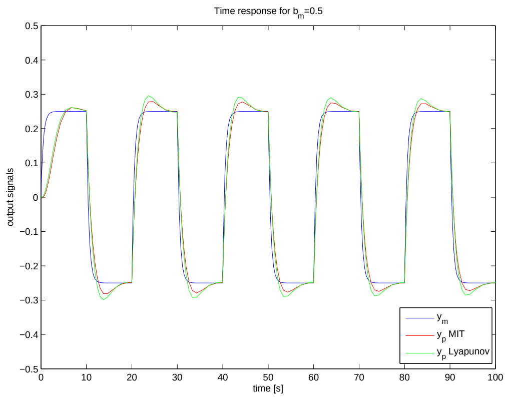

Figure 2.13: Time response for $b_m = 0.5$.

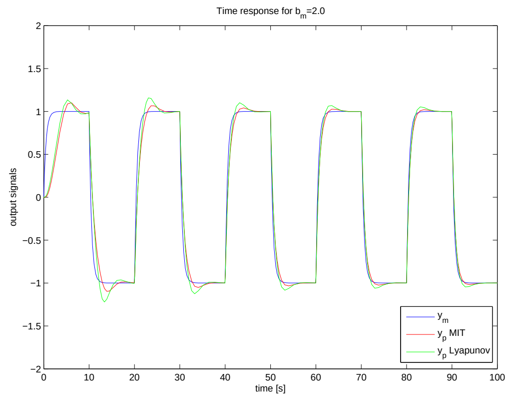

Figure 2.14: Time response for $b_m = 2.0$.

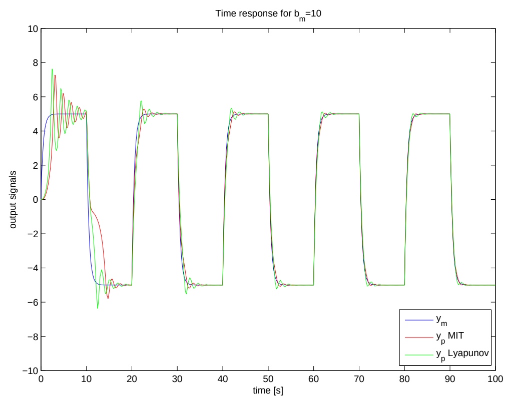

Figure 2.15: Time response for $b_m = 10$

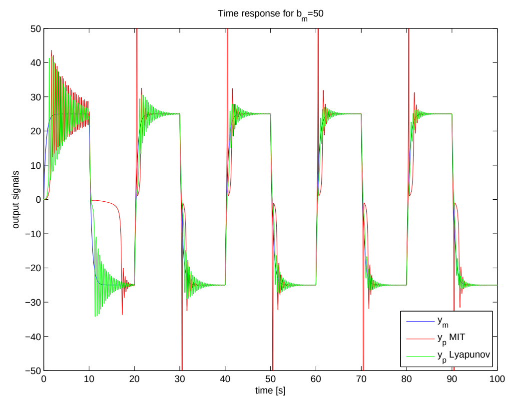

Figure 2.16: Time response for $b_m = 50$

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